The Empirical Econometrics and Quantitative Economics Letters
Volume 5, Number 2 (June 2017): pp. 23 – 32
ISSN 2286 – 7147

Modelling the growth of Internet users

Wiriyaporn Wonggattaleekam

[1] *Faculty of Economics, Chiang Mai University*
E-mail: w.wonggattaleekam@gmail.com

ABSTRACT

This paper aims at modelling the time-dependent characteristics of the internet user numbers. The model employs the stochastic exponential growth in which the growth uncertainty is represented by a standard Brownian motion (sBm). The parameter identification is carried out using the Bayesian inference. The Markov Chain Monte Carlo (MCMC) is used as a computational algorithm for the Bayesian inference. The results from the proposed methodology facilitate the future planning of economic related to the Internet usage, including the infrastructure preparation.

Keywords: Internet User Number Growth, Stochastic Exponential Model, Bayesian Inference, Economic Planning

JEL Classification: C11, C73, L86

1. Introduction

Communication is crucial for human beings for both personal and commercial reasons. (Deak, 2004) Additionally, human communication has evolved for several centuries and can be divided into six stages. First, a verbal language which is the language wildly spoken by human beings to come up with an identical understanding. Second, consequently, the writing using the symbols was initiated into the portrayals of words and thoughts. (Smithsonian National Museum of Natural History, 2016) Taken together, the verbal and symbolic written languages are the enrichment of the state-of-the-art human life. Third, roughly in 1439, Johannes Gutenberg, German, invented the movable type printing technology using the printing press. From that point forward, there were officially portable reading materials or printed books available and spreading all over the world. (Junior Scholastic, 2009) Fourth, it came to an introduction of the telegraph, a contacting via a message with someone from the distance. Additionally, in 1876, Alexander Graham Bell firstly invented a speaking telegraph, telephone, (Park, 1877) which the sound could be transmitted from one place to another place on a wire. (A+E Networks, 2009) Fifth, the wireless technology, which was invented by Nikola Tesla and Guglielmo Marconi (Clark, 2012), began to be applied in communications. Surprisingly, unlike the original and speaking telegraph, it could transmit text messages, images and voice mails. (West, 2012) Focusing mainly on the history of wireless mobile telephony development, there are five generations: 1G, 2G, 3G, 4G and 5G respectively. To summarise, the higher generation the mobile telecommunication is, the bigger and faster the data and information are able to be transmitted. Last, it has come to the Internet and World Wide Web era and both, in turn, illustrate the term: digital economy. Tech Target (2016) has given the definition of digital economy: the worldwide economic transaction network which is allowed by Information and Communications Technologies (ICTs). In simpler terms, the digital economy is the economy depending on the digital technologies. What's more, the OECD digital outlook report (2015) stated that ICTs have integrated with ranking from profession to personal lifestyle. The ways social interactions and personal relationships undertake have changed taking ICTs into account comparing to the old days. People can be online anywhere that the internet is available: in public, at workplace, at home or even on the move.

The digital economy has been growing so fast and affects variety sectors in the economies, i.e., entertainment and media, tourism, hospitality, education, publishing, banking, transportation, retail, energy, health, etc. (British Computer Society, n.d.) Furthermore, Deak (2004) declared that ICT has killed distance. In other words, it makes the national, regional, continental and global connections more possible for individuals, businesses, and governments. The ICTs, not to mention the Internet, have created several economic products: to name a few, the increase in the inflow foreign direct investment (FDI) (Choi, 2002), the decline of inflation rate (Yi and Choi, 2005), the determinant of the tourist arrivals (Naudé and Saayman, 2005) and the positive impacts on the economic growth (Sassi and Goaied, 2013; Salahuddin and Gow, 2015).

Focusing on the internet usage, the annually global Internet user numbers, grow increasingly every year .Simultaneously, in Thailand, the numbers of Internet users have climbed continuously year by year. (World Bank, 2016) Importantly, there are many existing factors influencing the increase in numbers of Internet users or the penetration of Internet use. Namely, in case of the mobile internet usage, Srinuan, et al. (2011) researched the determinate factors involving the consumer's mobile Internet accession.

The findings revealed that price, availability of fixed-line phone, age and area of living gave the strongest determination to the mobile Internet use.

Thai government has been promoting the digital economy for years because the government believe that the digital economy increases the competitiveness of the Thai industrial sector. Moreover, it is also an elementary aspect to drive Thailand economy by encouraging the innovation and growth. (Royal Thai Government, 2015) Recently, Thailand's National Legislative Assembly approved the establishment of the Ministry of Digital Economy and Society which holds the mission of enhancing the country into a digital society. (Zeldin, 2016) Accordingly, the Ministry of Digital Economy and Society's committee approved the National Digital Economy Master Plan for Digital Thailand. The goal of this master plan is to provide the full potential of development for the telecommunications infrastructure, information system, innovations, etc. Furthermore, the plan spreads over 20 years and holds 6 strategies: Hard Infrastructure, Digital Economy Acceleration, Digital Society, Service Infrastructure, Digital Workforce and Soft Infrastructure. (Royal Thai Government, 2016) More interestingly, in the near future, the royal Thai government expects to make availabilities of the broadband service to every village and also the free Wi-Fi spots all over the country.

To add more points, nowadays, the digital technologies provides variety of benefits to the society. First, they help people save time and cost. For instance, people can shop on the online stores and the ordered goods are delivered at the doors. There is no more travelling to the store and some stores offer more discounts if the customers shop online. Another, they bring up the efficiency, for example, building one aircraft requires millions of parts. A very-well-known aircraft manufacturer, Airbus, has used the high technologies to communicate with the suppliers and among branches to manage the deliveries of aeroplane parts to the assembly lines correctly and on time, in turn, causes the production process and saves cost of holding extra parts at the warehouse, respectively. (Airbus, n.d.) Researchers have studied the effect of the Internet users, the Internet usage or the ICTs. A number of study analyses have examined the relationship between the Internet and a variety of economic consequences. Choi (2002) studied the impact of the Internet on the level of inflow foreign direct investment (FDI) with a hypothesis that the Internet improved the productivity and, in turn, increased the FDI. The finding showed that the Internet gave positive impact to the FDI volume. Moreover, Yi and Choi (2005) investigated the Internet on inflation. They found that the increase in the Internet users reduced the inflation rate.

Interestingly, Naudé and Saayman (2005) proposed the paper on tourism in Africa and found that the Internet usage was one of the determinants of the tourist arrivals. Furthermore, Salahuddin and Gow (2015) studied the effect of Internet usage on the economic growth in South Africa. The result showed that the Internet usage had not only the long-run relationship but also the positive impact on the economic growth. Similarly, Elgin (2012) examined the relationship between the internet usage volume and the size of the shadow economy. Then, it revealed that the increase in the volume of the Internet usage rose the shadow economy's size.

Equally important, Jung, et al. (2013) demonstrated the direct and indirect effects of technological convergence. The results revealed that the ICT had the directly positive effect on the labour productivity and the indirectly positive effect on total factor

productivity (TFP) growth rate. Sassi and Goaied (2013) studied about effect of financial development and ICT on the economic growth. They discovered that the ICT penetration gave the directly positive effect on the economic growth significantly, but the financial development itself gave the negative effect. More interestingly, the reaching of the ICT development threshold can improve the financial development.

The literature above shows the significant role of Internet usage as a means of economic promotion. However, the usage depends also the infrastructure. To prepare for the future expansion of the Internet usage, the modelling of growth in the Internet user numbers becomes necessary.

This paper introduces a stochastic exponential growth model to characterise the growth in the Internet user numbers. The model is an extension of the deterministic model, which reflects the inherent uncertainty. Therefore, the proposed model is more realistic then the deterministic one. The Bayesian inference is employed for identifying the model parameters. The proposed methodology is shown through an application with real data.

2. Methodology

To start with, the exponential growth model, which is the simplest and the indubitably central model for the population growth research, is proposed. Due to the uncertainties, it is more rationale that the growth rate is considered uncertain and that leads to the application of a stochastic process which takes the environmental effects into account. Accordingly, a stochastic exponential growth model is proposed in this work, in which the growth rate is in terms of Brownian motion.

The core model of this study is

$$U_t = U_0 \exp\left(\mu t - \frac{\sigma^2}{2} t + \sigma B_t\right) \quad (1)$$

, where

U_t is the number of Internet users at time t.
U_0 is the initial number at time $t = 0$.
μ is the rate of change of the number of Internet users.
σ is the volatility.
B_t is one-dimensional Brownian motion.

Next, there are two main parts for this session: the model identification and the prediction using the Bayesian and Monte Carlo methods, not to mention the path generation, respectively.

Bayesian method is proposed to estimate the parameters, μ and σ in model (1). Bayesian inference can be obtained in the form of the posterior distribution, which is of the multiplication of likelihood function to the prior (Bishop, 2009), from the Bayes theorem. (Harnpornchai & Autchariyapanitkul, 2016)

The posterior distribution's form is

$$f(\mu, \sigma|D) \propto L(D|\mu, \sigma)\, f(\mu, \sigma) \quad (2)$$

, where

$f(\mu, \sigma|D)$ is the posterior distribution.
$L(D|\mu, \sigma)$ is the likelihood function.
$f(\mu, \sigma)$ is the prior.
D is the research data.

The structure of D is

$$D = \left(t_1, Y_{t_1}\right), \dots, \left(t_N, Y_{t_N}\right) \tag{3}$$

The likelihood function takes the structure:

$$L(D|\mu, \sigma) = \frac{1}{(2\pi)^{N/2}|\Sigma|^{1/2}} \exp[-Z^T \Sigma^{-1} Z] \tag{4}$$

,where

N is the numbers of data used in the inference.

and

$$Z_k = \frac{1}{\sigma}\left(Y_{t_k} - \mu t_k - \frac{\sigma^2}{2} t_k\right); k = 1, \dots, N \tag{5}$$

and

$$\Sigma_{ij} = E\left[B_{t_i} B_{t_j}\right] = \frac{1}{2}\left(t_i + t_j - |t_i - t_j|\right); i, j = 1, \dots, N \tag{6}$$

According to the inferred parameter using the Bayesian method, the numbers of Internet users at time t can be probabilistically obtained.

The Markov Chain Monte Carlo (MCMC) method is conducted to determine the posterior (2). (Harnpornchai & Autchariyapanitkul, 2016)

Given

$\theta = [\theta_1 \ \dots \ \theta_p]$ is a vector of the parameters $\theta_j \ ; j = 1, \dots, p$

,where

p is the total number of the parameters to be calculated.

The method runs as follow
1. Draw a individual θ_{l+1} from $q(\theta_{l+1}|\theta_l \ ; l = 1, \dots, M)$.
2. Accept the individual θ_{l+1} with the probability of $\alpha(\theta_l, \theta_{l+1})$; otherwise set $\theta_{l+1} = \theta_l$.

Therefore, the probability of acceptance is

$$\alpha(\theta_l, \theta_{l+1}) = \min\left(\frac{\pi(\theta_{l+1})/q(\theta_{l+1}|\theta_l)}{\pi(\theta_l)/q(\theta_l|\theta_{l+1})}, l\right) \tag{7}$$

The independent density, regard a PDF, the next individual does not depend on the current one: $q(\theta_{l+1}) = q(\theta_{l+1})$.

According to the independent density mentioned above, the probability of acceptance is
1. Draw a individual θ_{l+1} from $q(\theta_{l+1})$.
2. Accept the individual θ_{l+1} with the probability of $\alpha(\theta_l, \theta_{l+1})$.
Again, the probability of acceptance is

$$\alpha(\theta_l, \theta_{l+1}) = \min\left(\frac{\pi(\theta_{l+1})/q(\theta_{l+1})}{\pi(\theta_l)/q(\theta_l)}, l\right) \quad (8)$$

3. Data

To begin with, the data in this study are the secondary data which were gathered via the National Electronics and Computer Technology Centre; NECTEC (NECTEC, 2016) of Thailand. Plus, the research data are yearly Internet user numbers collected from 1991 to 2015, see Figure 1.

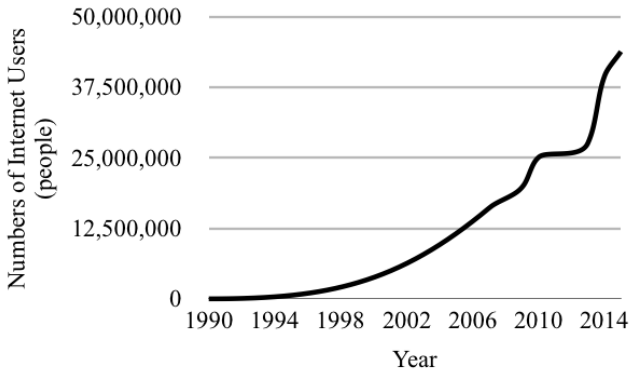

Figure 1: Numbers of Internet Users in Thailand from 1990 to 2015
Soruce: NECTEC, 2016

4. Results

By inference, the priors are as follow

$$f(\mu, \sigma) = f(\mu)f(\sigma) \quad (9)$$

, where

$$f(\mu) \sim N(0.33,1) \quad (10)$$

$$f(\sigma) \sim INVGAMMA(1,1) \quad (11)$$

The normal distribution was applied to the μ prior. In contrast, as σ cannot be negative. Therefore, the inverse gamma distribution, which has non-negative property, was applied to the σ prior. Furthermore, the complete number of realisations was accounted for 1,000,000 with the 100,000-realisation burn-in. In addition, the histograms of the posterior distribution of parameters μ and σ was illustrated in Figure 2 and Figure 3, respectively.

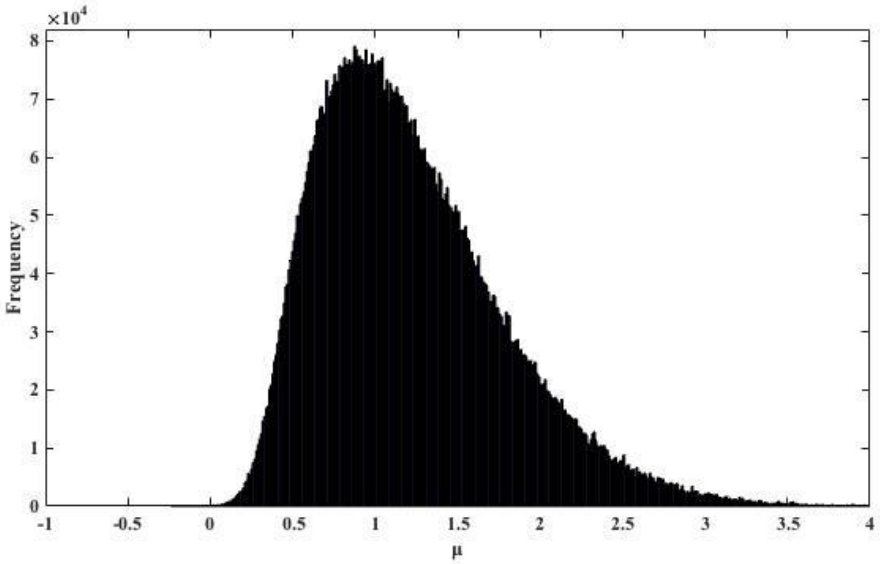

Figure 2: Histogram of μ with 100,000-realisation burn-in
Source: Own Illustration

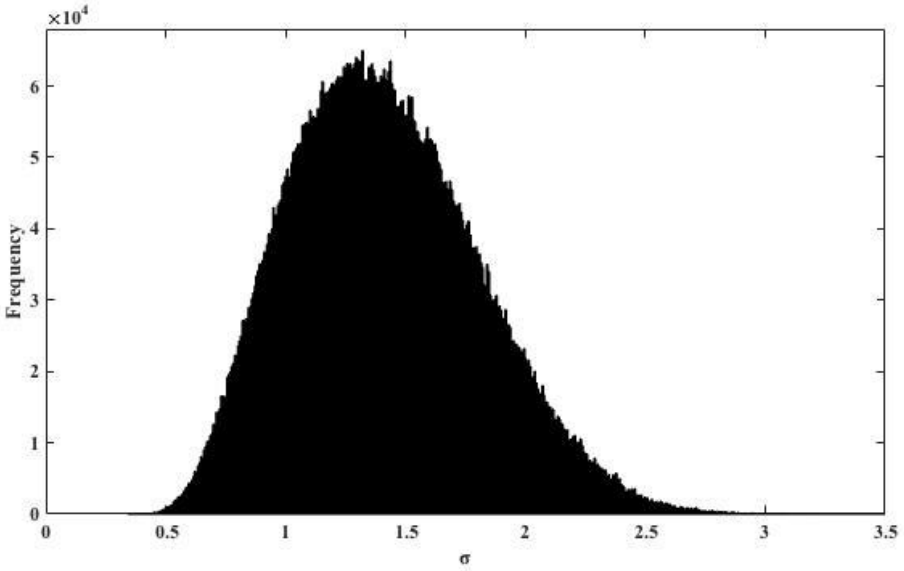

Figure 3: Histogram of σ with 100,000-realisation burn-in
Source: Own Illustration

The mean is applied as the estimate. Accordingly, the parameters $\hat{\mu}$ and $\hat{\sigma}$ are identified as follow

$$\hat{\mu} = 1.2147 \quad (12)$$

$$\hat{\sigma} = 1.4112 \quad (13)$$

The convergence diagnostics of $\hat{\mu}$ and $\hat{\sigma}$ are certified through plotting the tracing, running-mean and autocorrelation function (ACF) in Figure 4 and Figure 5, respectively.

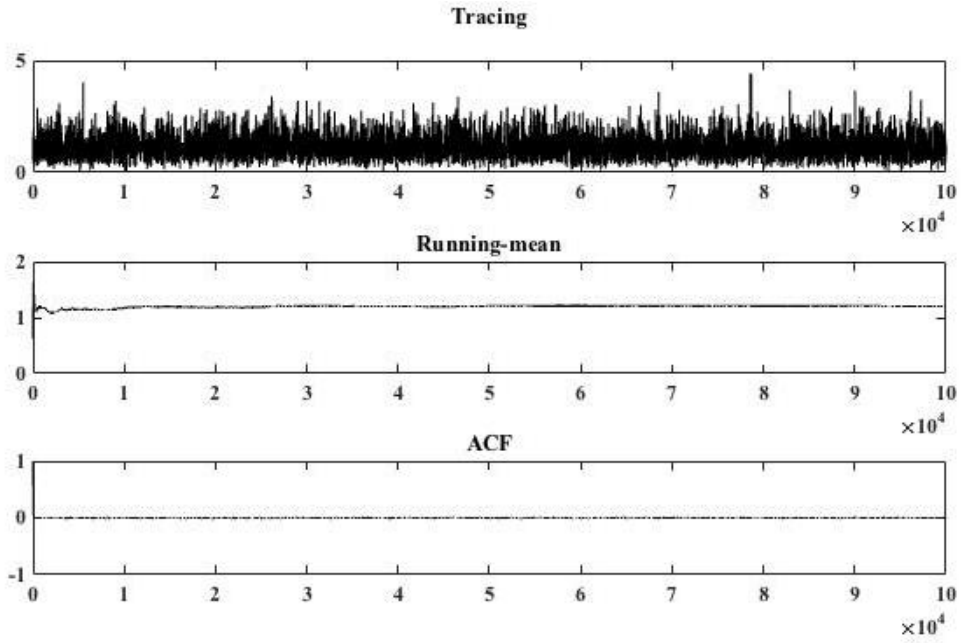

Figure 4: Convergence diagnostics of $\hat{\mu}$ via plotting the tracing, running-mean and autocorrelation function (ACF)
Source: Own Illustration

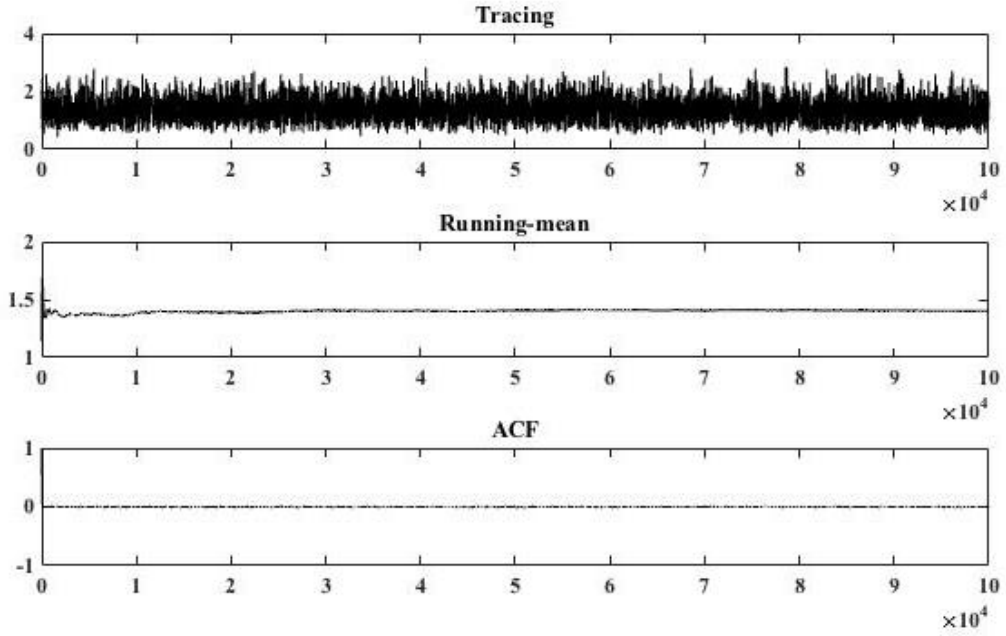

Figure 5: Convergence diagnostics of $\hat{\sigma}$ via plotting the tracing, running-mean and autocorrelation function (ACF)
Source: Own Illustration

After the substitution of the identified values, $\hat{\mu}$ and $\hat{\sigma}$, the core model of this study is

$$U_t = U_0 \exp\left(1.2147t - \frac{(1.4112)^2}{2}t + 1.4112B_t\right) \quad (14)$$

5. Conclusions

Internet plays an important role of economic promotion. To prepare for the future expansion of the Internet usage, the modelling of growth in the Internet users becomes necessary.

This paper introduces a stochastic exponential growth model to characterize the growth in the Internet users. The proposed model is more realistic then the deterministic one in that the uncertainty is included into the model. The Bayesian inference is employed for identifying the model parameters. The results from the inference make the model ready for the future forecasting.

REFERENCES

A+E Networks. 2009. "Alexander Graham Bell patents the telephone," Retrieved from http://www.history.com/this-day-in-history/alexander-graham-bell-patents-the-telephone

Airbus. n.d. "Innovation," Retrieved from http://www.airbus.com/innovation/

Bishop, C. 2009. "Introduction to Bayesian Inference – Part 1," Retrieved from http://mlg.eng.cam.ac.uk/mlss09/mlss_slides/Bishop-MLSS-09-1.pdf

British Computer Society. n.d. "The Digital Economy," Retrieved from https://policy.bcs.org/sites/policy.bcs.org/files/digital%20economy%20Final%20version_0.pdf

Choi, C. 2002. "Does the Internet stimulate inward foreign direct investment?," Journal of Policy Modeling, 25 (2003), pp.319-326. Retrieved from http://www.sciencedirect.com/science/article/pii/S0161893802002028

Clark, J. F. 2012. "History of Mobile Telephony," Retrieved from http://www.uky.edu/~jclark/mas490apps/History%20of%20Mobile%20Telephony.pdf

Deak, E.J. 2004. "The Economics of e-Commerce and the Internet," Ohio: South-Western.

Elgin, C. 2012. "Internet usage and the shadow economy: Evidence from panel data," Economic Systems, 37 (2013), 111–121. Retrieved from http://www.sciencedirect.com/science/article/pii/S0939362512001069

Harnpornchai, N., & Autchariyapanitkul, K. 2016. "Modeling stock market dynamics with stochastic differential equation driven by fractional Brownian motion: A Bayesian method," Thai Journal of Mathematics.

Jung, H., Na, K. & Yoon, C. 2013. "The role of ICT in Korea's economic growth: Productivity changes across industries since the 1990," Telecommunications Policy, 37 (2013), 292–310. Retrieved from http://www.sciencedirect.com/science/article/pii/S0308596112001115

Junior Scholastic. 2009. "How Did the Printing Press Change History?," Retrieved from http://teacher.scholastic.com/scholasticnews/magazines/junior/pdfs/JUNIOR-033009-REPRO-01.pdf

Naudé, W. & Saayman, A. 2005. "Determinants of tourist arrivals in Africa: a panel data regression analysis," Tourism Economics, 11 (3), 365–391. Retrieved from https://mpra.ub.uni-muenchen.de/16479/

NECTEC. 2016. "Internet Users," Retrieved from http://internet.nectec.or.th/webstats/internetuser.iir?Sec=internetuser

OECD. 2015. "OECD Digital Economy Outlook 2015," Retrieved from http://ec.europa.eu/eurostat/documents/42577/3222224/Digital+economy+outlook+2015/dbdec3c6-ca38-432c-82f2-1e330d9d6a24

Park, M. 1877. "Speaking Telegraph," Retrieved from http://edison.rutgers.edu/yearofinno/TAEBdocs/Doc863_Telephone_2-12-77.pdf?DocId=MBN002033

Royal Thai Government. 2015. "Digital Economy," Retrieved from http://www.thaigov.go.th/index.php/en/issues/item/93149-93149.html

Royal Thai Government. 2016. "Committee for Digital Economy and Society approves National Digital Economy Master Plan," Retrieved from http://www.thaigov.go.th/index.php/en/government-en1/item/99989-99989.html

Salahuddin, M. & Gow, J. 2015. "The effects of Internet usage, financial development and trade openness on economic growth in South Africa: A time series analysis," Telematics and Informatics, 33 (2016), 1141–1154. Retrieved from http://www.sciencedirect.com/science/article/pii/S0736585315001331

Sassi, S. & Goaied, M. 2013. "Financial development, ICT diffusion and economic growth: Lessons from MENA region," Telecommunications Policy, 37 (2013), 252–261. Retrieved from http://www.sciencedirect.com/science/article/pii/S0308596112002054

Smithsonian National Museum of Natural History. 2016. "Human Characteristics**:** Language and Symbols,**"** Retrieved from http://humanorigins.si.edu/human-characteristics/language-symbols

Srinuan, C., Srinuan, P. & Bohlin, E. 2011. "An analysis of mobile Internet access in Thailand: Implications for bridging the digital divide," Telematics and Informatics, 29 (2012), 254–262. Retrieved from http://www.sciencedirect.com/science/article/pii/S0736585311000694

Tech Target. 2016. "Definition Digital Economy," Retrieved from http://searchcio.techtarget.com/definition/digital-economy

West, N. 2012. "The 10 Inventions of Nikola Tesla That Changed the World," Retrieved from http://www.activistpost.com/2012/01/10-inventions-of-nikola-tesla-that.html

World Bank. 2016. "Internet users **(**per 100 people**),"** Retrieved from http://data.worldbank.org/indicator/IT.NET.USER.P2?view=chart

Yi, M. & Choi, C. 2005. "The effect of the Internet on inflation: Panel data evidence," Journal of Policy Modeling, 27 (2005), 885–889. Retrieved from http://www.sciencedirect.com/science/article/pii/S0161893805000906

Zeldin, W. 2016. "Thailand**:** Digital Ministry Established as Part of National Digital Economy Plan," Retrieved from http://www.loc.gov/law/foreign-news/article/thailand-digital-ministry-established-as-part-of-national-digital-economy-plan/