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# Mathematical model of impossibility of sharing economy under endless game with two agents

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## ABSTRACT

This paper shows mathematical reasoning to reveal the natural rule that prevent sharing economy to occur. The model consists of two sides. When the first side has power over the other in the first place, he maximizes his utility by gaining all the benefits that can be divided to the other. After the switching of the administrative position which empowers the second side to take over the power, the only equilibrium is that the second side gains all the benefits that can be shared too. When this game repeats endlessly, the model predicts no sharing of benefits between both sides. Therefore, it reveals the impossibility of sharing economy under this certain condition.

*Keywords:* Abuse of power, revenge, sharing economy, repeated game, competitive equilibrium

*JEL Classification:* C71, C72, C73

## 1. Mathematical setting

The economy consists of two agents. Let agent A has his utility of $U_A$ which is equal to summation of benefits that he gains in period t. Agent B has the same type of utility, $U_B$. Benefits that agent A gains in period t consists of two parts, the benefit that belongs to only agent A and the benefit that can be divided to agent A and B. Benefits of agent B follows the same way. Summation of the second part of benefits that can be shared to agent A and B equals to unity.

$U_A = B_A + \alpha B_{AB}$
$U_B = B_B + \beta B_{AB}$
$\alpha B_{AB} + \beta B_{AB=1}$

The competition between agent A and B for the second part of the benefits is determined by the decision of the superior in terms of administrative power in period t. Agent A and B decide to toast a coin to decide who will gain the administrative power in the first place. Then after the certain period, the other agent will take over the power. The turns will be consistent in this way endlessly.

## 2. Equilibrium

Once agent A gets a luckier result of toasting the coin, he gets the administrative power first. His decision is to divide the second part of benefits that can be shared between both agents. The equilibrium that yields maximized utility is to take all of that benefit, leaving only the first part of benefits which cannot be transferred to him to agent B.

Max $U_A = B_A + B_{AB}$

Unfortunately, the period that agent A is superior to agent B has time limit. When agent B takes over the administrative power, he will do the same without sharing the second part of benefits to agent A.

Max $U_B = B_B + B_{AB}$

## 3. Long run equilibrium

In the long run, when agent A and B switch to gain the administrative power, the summation of long run benefits of each agent under the circumstance of administrator takes all equals to the summation of the long run benefits under sharing any portion of the second part of benefits to the other agent.

Without sharing a portion of the second part of benefits to agent B:

Long run $U_A = \sum_{t=1}^{t+2n}(B_A + B_{AB}) + \sum_{t=2}^{t+2n} B_A$

Long run $U_A = nB_A + ½n\ B_{AB}$

With sharing a portion of the second part of benefits to agent B:

Long run $U_A = \sum_{t=1}^{t+2n}(B_A + \alpha B_{AB}) + \sum_{t=2}^{t+2n}(B_A + (1-\alpha)B_{AB})$

Long run $U_A = nB_A + ½n\ \alpha B_{AB} + ½n\ (1-\alpha)B_{AB}$

Long run $U_A = nB_A + ½n\ B_{AB}$

**4. The equilibrium in the first round (Short run equilibrium)**

Recall the first period when agent A gains the administrative power first. When he knows that the long run benefit with and without sharing the second part of benefits to agent B are the same, he evaluates the short run benefit to make decision. It is clear that if he takes that whole portion of the second part of benefits, he will gain the most benefit than sharing any portion to agent B. This is the reason why agent A does not share anything to B although he can.

Short run $U_A$ without sharing is Max $U_A = B_A + B_{AB}$

Short run $U_A$ with sharing is $U_A = B_A + \alpha B_{AB}$

Therefore, this model predicts no sharing economy in the endless game of two agents.

**5. Conclusions**

It is clear that in the endless game of two persons, the sharing economy does not exist. This leads to the condemner of economics on the selfishness of human. It may imply that in order to avoid the selfishness through the society, the game must not be endless. It is interesting to prove whether after the assumption of endless game is replaced by finite game, the sharing economy can exist or not.

This paper does not support Buddhism economics as much as it is expected. Buddhism economics may assume endless game by the life after death and the rebirth. The thought prevents human to do wrong or bad things for fear that they would be punished later after death. However, by the mathematical proof in this paper, this thought may lead to selfishness in this life.

To argue against the selfishness in the present life, Buddhism economics may find the sacrifice in this life would be reward after death. This is an inverse of selfishness when a person does not take benefit in period one but expect more return in period n which will happen after death. This phenomenon is strange but famous in Thailand. Many temples encourage people to donate very much to invest in the future life after death. The temples promise luxury estate in heaven. It is also strange that the Thai and people from other countries who donated believe that. It may be worth to prove such kind of investment in the next paper.