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Currency substitution, inflation and economic growth in Nigeria: A simultaneous equations analysis

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ABSTRACT

The paper employs the two-stage least squares (2SLS) estimation technique to estimate a system of simultaneous equations for the purpose of investigating the interrelationships among currency substitution, inflation and economic growth in Nigeria. Using annual time series data covering the period from 1994 to 2013, the study finds that (1) the effects of inflation and real GDP growth rate on currency substitution are not statistically significant (2) depreciation of the domestic currency increases the rate of currency substitution (3) investment helps reduce currency substitution (4) inflation was positively associated with the growth of real GDP within the sample period. This was attributed to the fact that inflation in most part of the sample period was less than the threshold identified in the literature. (5) currency substitution contributes to the growth of real GDP (6) increase in government expenditure within the period adversely affected real GDP growth (7) currency substitution and investment help reduce inflation (8) the depreciation of the local currency engendered increase in inflation. The implications of these findings for policy are discussed

Keywords: Currency Substitution, Inflation, Economic Growth, Exchange Rate, simultaneous equation
JEL Classification: E41, E31, C32

1. **Introduction**

Currency substitution refers to the use of a foreign currency to facilitate economic transactions (and to preserve wealth) in an economy. It could be partial or full currency substitution. Partial currency substitution (also referred to as unofficial or *de facto* currency substitution) is the use of a foreign currency alongside or in parallel to the domestic currency for economic transactions in a country. The foreign currency constitutes parts of the domestic media of payment (Handa, 2000) but the foreign currency is not considered a legal tender in the country. Full currency substitution (also known as official or *de jure* currency substitution) refers to absolute substitution of a foreign currency for the local currency for economic transactions in a country. The foreign currency becomes the sole legal tender of the country, thus undermining the independence of the monetary authority.

Several factors engender the use of foreign currency in an economy and most of these factors are also affected by currency substitution. The factors include inflation, the exchange rate of the domestic currency, growth rate of the economy, etc. Inflation enhances the desirability of a foreign currency of higher value as it reduces the purchasing power or value of the local currency. This leads to increase in demand for the currency as a way of hedging against the risk of loss of value of the domestic currency if inflation is expected to persist, thus fuelling currency substitution in the economy. Exchange rate which is an indication of the strength or value of currency, also determines the rate of currency substitution in an economy. Increase in exchange rate of the domestic currency (i.e. depreciation) reduces the attractiveness of the local currency vis-à-vis a foreign currency (of higher value), while currency appreciation in theory, reduces the attractiveness of the foreign currency if the appreciation is expected to be sustained. High rate of currency substitution in an economy is much likely to further depreciate the local currency. The rate of growth of an economy also affects the chances of currency substitution in the economy. Currency substitution is more prevalent in countries with underdeveloped or developing economies, and whose currency is highly depreciated. The degree of currency substitution in highly developed economies with strong currencies and impressive growth rate is quite low.

Just as interrelationships exist between currency substitution and the macroeconomic variables identified above, interrelationships also exist among the variables themselves. In the absence of barriers to cross border trade, inflation increases demand for imports (some of which may be relatively cheaper than locally made goods) and in the absence of sufficient reserves to back up or support the value of the local currency by the government through the sale of the foreign currency in the domestic foreign exchange market, this engenders depreciation of the domestic currency. On the other hand, international trade theory suggests that increase in the exchange rate (i.e. currency depreciation or devaluation depending on whether the exchange rate regime is flexible or fixed) boosts export as it increases the domestic market price of foreign goods, thus reducing the demand for them. The import substitution brought about by currency depreciation engenders the release of more resources (if they exist) towards the production of the affected import commodity. This is expected to lead to expansion in output and reduction in prices. This however is in theory. However, for an import dependent small open economy, currency depreciation likely engenders inflation as this causes the domestic market prices on imported commodities to increase.

Hitherto, inflation was generally thought of as adversely affecting the growth rate of an economy. However, several empirical works such as those of Bawa and Abdullahi (2012), Danladi (2013), Doguwa (2013), etc. have shown that inflation only negatively affects economic growth beyond some threshold As a matter of fact, the threshold for Nigeria is

estimated at 10% - 13.5%. This is to say that inflation could have positive or no effect on economic growth below the threshold. Moreover, considering that inflation is inversely related to unemployment according to the Philips curve phenomenon, reduction in unemployment arising from some level of inflation, could positively affect economic growth considering that positive relationship exists between employment and growth. Economic growth theoretically reduces inflation as a result of expansion in output it connotes.

The relationship between exchange rate and economic growth has been much investigated and the outcomes of the investigations are well documented in the extant literature. International trade theory suggests that currency depreciation boost exports, and the Export-led Growth (ELG) hypothesis suggests that export expansion is a vital ingredient or engine of economic growth. On the other hand, economic growth also affects the exchange rate of the domestic currency. Higher economic growth rate or output expansion, tends to strengthen to domestic currency (i.e. reduces the rate at which it exchanges for other currencies, particularly those of its trading partners) as the demand for import reduces, while output and export expand.

The objective of this paper is to empirically investigate the relationships among currency substitution (measured as the ratio of foreign currency deposit in the commercial banks to broad money supply, M2), inflation and economic growth in Nigeria in the period from 1994 to 2013. Its major contribution to the literature is the identification of the interrelationships among these variables using a system of simultaneous equations estimated with the two-stage least squares estimation technique instead of assuming a uni-directional relationship, thus avoiding the problem of simultaneous equations bias or the least squares bias which could have been encountered if the traditional ordinary least squares estimation technique was employed. Empirical works by Adom, Sharma and Morshed (2006) and Doguwa (2014) give evidence of currency substitution in Nigeria. Investigating the interrelationships between currency substitution, inflation and economic growth in the country will therefore not be out of place.

2. **Literature Review**

Adom, Sharma and Morshed (2009) investigates the presence of currency substitution in eight African countries (Egypt, Ghana, Kenya, Morocco, Nigeria, South Africa, Tunisia and Zambia) in the period from 1976 to 2005, using regional and the U.S. Dollar as anchor currencies. The study reveals *inter alia*, prevalence of currency substitution in Ghana and Nigeria when the Franc was used as the anchor currency, but when the U.S. Dollar was used as the anchor currency, currency substitution was observed to be prevalent in Nigeria. This tends to suggest that high level of foreign currency usage in Nigeria for economic transactions.

Yinusa (2009) examined the role of fluctuations in macroeconomic variables in the dollarization of deposit in 18 sub-Sahara African countries in the period from 1980-2004, using the standard money demand model for dollarization in small open economies. The study finds that inflation, expectations about changes in exchange rate and, interactions between capital accounts restrictions and domestic inflation significantly explain deposit dollarization in sub-Sahara Africa.

Akinlo (|2003) investigates the effect of the depreciation of the Naira on currency substitution in the period from 1980 to 2000. The investigation reveals that the depreciation of the currency does not engender currency substitution, but rather engenders increase in the wealth of holders of foreign currency denominated accounts as they convert the currency whose value is higher to the local one whose value has depreciated. Sawada and Yotopoulos

(2000) investigate the relationships between currency substitution, speculation and financial crisis. The study shows *inter alia* that high degree of currency substitution engenders devaluation of the local currency.

Doguwa (2014) examines the existence, causes and effects of currency substitution (measured as foreign currency/naira demand deposit ratio) in Nigeria by estimating conventional money demand equations based on partial adjustment and autoregressive distributed lag models. The analysis gives strong evidence of currency substitution in the country. It further shows that currency substitution in Nigeria has been influenced by expectations of currency devaluation, exchange rate risk and some macroeconomic policies implemented since the Yar'Adua-Jonathan presidency. As a matter of fact, the study shows that policies implemented since the beginning of the Yar' Adua-Jonathan presidency have led to higher degree of currency substitution in Nigeria.

Fielding and Shorthand (2002) employs regression analysis (ordinary least squares) and maximum likelihood estimation to investigate the effect of political violence on currency substitution in Egypt. The study finds, *inter alia*, that an increase in political violence will engender instantaneous switch from domestic currency assets to foreign currency assets. In other words political violence causes increase in the rate of currency substitution.

Asel (2009) investigates the effect of currency substitution on the economies of Asian Central using a model with money-in-the utility-funtion to estimate the elasticity of substitution between domestic and foreign currencies in three Asian Central Countries – Kazakhstan, the Kyrgyz republic and Tajikistan. The analysis indicates that though currency substitution decreases governments' seigniorage revenue, holding foreign currency can enhance welfare if the domestic currency depreciates relative to the foreign currency in which residents hold their wealth.

3. Model and Estimation Methodology

The paper employs the 2SLS simultaneous equations estimation technique to investigate the relationships between currency substitution, inflation and economic growth In Nigeria. We begin by specifying the system of equations functionally as:

$$CS = f(INF, INF(-1), RGDPG, EXRT, INV(-1)) \quad (1)$$

$$RGDPG = f(CS, CS(-1), INF, INV(-1), GOVEXP, TOPEN) \quad (2)$$

$$INF = f(CS, INV(-1), GOVEXP, EXRT(-1), TOPEN) \quad (3)$$

Where:

CS = Currency Substitution

INF = Inflation

RGDPG = Real GDP growth rate, proxy for economic growth

EXRT = Exchange rate

GOVEXP = Federal Government total expenditure

TOPEN = Trade openness

INF (-1) = One year lagged value of inflation

INV (-1) = One year lagged value of investment

CS (-1) = One year lagged value of currency substitution

EXRT (-1) = One year lagged value of exchange rate.

The empirical model is specified using the logarithm of the variables as:

$$LCS_t = a_0 + a_1LINF^+_t + a_2LINF^+_{t-1} + a_3LRGDPG^-_t + a_4LEXRT^+_t + a_5LINV^-_{t-1} + u_{1t} \quad 1^*$$

$$LRGDPG_t = b_0 + b_1LCS_t^{\pm} + b_2LCS_{t-1}^{\pm} + b_3LINF_t^{\pm} + b_4LINV_{t-1}^{+} + b_5LGOVEXP_t^{+} + b_6LTOPEN_t^{+} + u_{2t} \quad 2^*$$

$$LINF_t = c_0 + c_1LCS_t^{+} + c_2\,LINV_{t-1}^{-} + c_3LGOVEXP_t^{-} + c_4LEXRT_{t-1}^{+} + c_5LTOPEN_t^{+} + u_{3t} \quad 3^*$$

The u_i's are error terms, which are not correlated with any of the exogenous variables in each equation. The a priori expectations are superscripted on the variables.

The system of simultaneous equation above has three endogenous variables (CS, INF and RGDPG). The other variables are exogenous variables. The first model of the system explains currency substation, while the second and third explain economic growth and inflation respectively. The test for endogeneity or simultaneity among the endogenous variables is conspicuously skipped (owing to the small size of the samples (data) used for the analysis), as that test is valid only for large samples (Gujarati and Porter, 2009).

Before proceeding to estimating the structural parameters of the model, we examine its identification status, as this is a compulsory condition for estimation. A non-identified or unidentified system of simultaneous equations cannot be estimated. To examine the identification status of the system, we first employ the *order condition* for identification. The result is presented in the Appendix. It is observed that all the equations of the system are over identified. Thus the system is over-identified. However, the *order condition* for identification is only a necessary condition for identification. To be quite certain of the identification status of the model, the *rank condition* is also employed. The results also shown in the Appendix indicate that the equations of the system are identified. Thus the entire system is identified, and consequent upon this, it can be estimated. The two-stage least squares (2SLS) estimation method shall be used to estimate the parameters of the model as this will yield estimates that are consistent and unbiased. This method of estimating a system of simultaneous equations involves using OLS to estimate the reduced form of each equation in the first stage to obtain estimated values of the endogenous variables in the first stage. In the second stage, the estimated values of the endogenous variables are substituted for their actual/observed values in each of the equations of the system and the equations are estimated using the OLS estimation technique to obtain estimates of the structural parameters. This ensures that the explanatory variables are not correlated with the residuals or error terms (u_i's), and that the estimated parameters are efficient and consistent. The results and their implications are presented in sections 5 and 6 respectively. Microfit 5 econometric package was used for the estimations.

4. **Data**

Data used for the estimations are annual time series data covering the period from 1994 to 2013. (The small sample size of the variables actually necessitated the skipping of the test for endogeneity or simultaneity). The study is limited to this period because data on foreign currency deposits in Nigeria's commercial banks begins from 1994. Source of all data for the analysis is the Central Bank of Nigeria Statistical Bulletin (2013).

5. **Results**

The results of the 2SLS estimations of the structural parameters of the models are presented in Tables 1(a), 1(b) and 1(c).

TABLE 1(a). Equation Explaining Currency Substitution

Two Stage Least Squares Estimation

Dependent variable is LCS with 19 observations used for estimation from 1995 to 2013

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
C	-2.0962	.73923	-2.8357[.014]
LINF	.034054	.12472	.27305[.789]
LINF(-1)	.10046	.059517	1.6879[.115]
LRGDPG	-.041996	.32113	-.13078[.898]
LEXRT	.93019	.18869	4.9297[.000]
LINV(-1)	-.11646	.027116	-4.2949[.001]

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R-Squared	.92412	R-Bar-Squared	.89493
S.E. of Regression	.27212	F-Stat. F(5,13)	31.6631[.000]
Mean of Dependent Variable	1.8795	S.D. of Dependent Variable	.83951
Residual Sum of Squares	.96266	Equation Log-likelihood	1.3739
DW-statistic	1.7016	System Log-likelihood	-4.7579
System AIC	-23.7579	System SBC	-32.7301

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Source: Author's estimations output from Microfit 5.0 time series econometric software

TABLE 1(b). Equation Explaining Economic Growth (Real GDP growth)

Two Stage Least Squares Estimation

Dependent variable is LRGDPG with 19 observations used for estimation from 1995 to 2013

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
C	1.0052	1.0649	.94390[.364]
LINF	.17062	.043932	3.8838[.002]
LCS	-.23817	.12636	-1.8848[.084]
LCS(-1)	.57374	.085434	6.7155[.000]
LINV(-1)	.0078657	.019476	.40387[.693]
LGOVEXP	-.048988	.021852	-2.2418[.045]
LTOPEN	-.061316	.26593	-.23057[.822]

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R-Squared	.95652	R-Bar-Squared	.93478
S.E. of Regression	.13199	F-Stat. F(6,12)	43.9966[.000]
Mean of Dependent Variable	1.6015	S.D. of Dependent Variable	.51683
Residual Sum of Squares	.20906	Equation Log-likelihood	15.8812
DW-statistic	1.9853	System Log-likelihood	-4.7579
System AIC	-23.7579	System SBC	-32.7301

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Source: Author's estimations output from Microfit 5.0 time series econometric software

TABLE 1(c). Equation Explaining Inflation

Two Stage Least Squares Estimation

Dependent variable is LINF with 19 observations used for estimation from 1995 to 2013

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
C	-7.2176	8.2459	-.87530 [.397]
LCS	-2.3094	.71448	-3.2323 [.007]
LINV(-1)	-.22804	.12896	-1.7684 [.100]
LGOVEXP	-.033957	.17379	-.19539 [.848]
LEXRT(-1)	1.7182	.81446	2.1096 [.055]
LTOPEN	1.8490	1.8191	1.0164 [.328]

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R-Squared	.43338	R-Bar-Squared	.21546
S.E. of Regression	.93182	F-Stat. F(5,13)	1.9886 [.147]
Mean of Dependent Variable	2.3386	S.D. of Dependent Variable	1.0520
Residual Sum of Squares	11.2878	Equation Log-likelihood	-22.0130
DW-statistic	1.7329	System Log-likelihood	-4.7579
System AIC	-23.7579	System SBC	-32.7301

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Source: Author's estimations output from Microfit 5.0 time series econometric software

6. **Discussions**

The equation explaining currency substitution (Table 1(a)) shows that the signs on all the variables conform to *a priori* expectations and that the effects of current and one-period lagged values of inflation on currency substitution is not statistically significant at the 5% and 10% levels. The effect of Real GDP growth is also not statistically significant. This suggests that inflation and economic growth are not strong determinants of the rate of currency substitution in the country. The effect of currency depreciation (that is, increase in the exchange rate of the Naira), on currency substitution is positive and strongly statistically significant (even at the 0.1% level). This indicates that the depreciation of the Naira contributes significantly to the increase in the rate of currency substitution in the country. This is not unexpected, as the depreciation of the domestic currency reduces its value and attractiveness, and enhance the relative value of the foreign currency (in this case the Dollar) making it more attractive. However, investment in the country helps reduce the rate of currency substitution therein. Investment affects currency substitution with a one-year lag. The effect is strongly statistically significant even at the 0.1% level. This could be attributed to the expansion in output which it engenders, which *ceteris paribus*, in turn has positive effect on export and negative effect on import and enhances the value of the local currency, thereby reducing the rate of currency substitution the country.

An examination of the diagnostic statistics reveals that the model has a good fit as shown by the R-squared which indicates that over 92 % of the systematic variation in the dependent variable is explained by the explanatory variables. This is just by the way, because the R-squared actually has no statistical meaning in 2SLS. The F-statistics which is strongly

significant also shows that the regressors are jointly significant in explaining variations in the dependent variable. The D.W. Statistics of 1.7 is satisfactory.

The policy options suggested by the results in Table 1a is that if the government desires to reduce the rate of currency substitution in the economy, she should consider channeling efforts at enhancing the value of the local currency, and putting measures in place to expand the level of investment (domestic and foreign) therein, such as creating a conducive ambience for businesses to thrive, design and implementation of appropriate macroeconomic policies, infrastructural development, reduction in cost of doing businesses, etc.

The equation explaining economic growth (Table 1(b)) reveals a positive and significant effect of inflation on economic growth in the country in the sample period. This suggests that inflation contributed positively to the growth of Nigeria's economy within the period under review. We are careful not to assert that this finding is contrary to expectations, as inflation could positively affect growth below some threshold inflation rate (as discussed previously). Empirical studies on threshold effects of inflation on economic growth Nigeria peg this threshold inflation in the 10-13.5% range. When it is considered that in most of the period under review, inflation rate was actually below the threshold, this result will not come as a surprise. Moreover, considering that inflation and unemployment are inversely related according to the Phillips Curve, the decline in unemployment (or increase in employment) associated with increase in inflation (not beyond the threshold) invariably translates into expansion in output. However, measures must be put in place to prevent hyperinflation, or escalation of inflation beyond the threshold.

The contemporaneous impact of currency substitution on economic growth is negative, but fails the test of statistical significance at the 5% level, though it passes the test at the 10% level, while the lagged impact is positive and strongly significant even at the 0.1% level. Thus the positive impact of CS on economic growth dominates the negative impact. There is therefore net positive one-period lagged impact of currency substitution on the growth of real GDP in Nigeria. This suggests that currency substitution impacts favourably on Nigeria's economic growth. This could be attributed to increase in the wealth of residents holding foreign currency denominated deposits (or balances) in the face of depreciation of the domestic currency.

Investment variable has the expected positive sign, but fails the test of statistical significance, an indication that investment in the country has not been sufficient to significantly drive the growth of the economy. The negative sign on government expenditure indicates that the growth in government expenditure (particularly in the democratic era, which has a preponderance of recurrent expenditure over capital expenditure) has had a depressing effect on the growth of the economy. This underscores the need for government to cut its expenditure and to make its expenditure more productive by spending more on capital projects. Trade openness variable has the opposite sign, and is statistically not significant – an indication that trade did not have any significant effect on the growth of real GDP in the sample period.

The model has a very high goodness of fit as shown by the R-squared and R-bar-squared. The highly significant F-statistics indicates that the explanatory variables are jointly significant in determination of the dependent variables. The DW-statistics indicates absence of first order serial correlation in the model. Thus the model is appropriate and can be relied upon for policy.

Finally, the equation explaining inflation (Table 1(c)) shows that currency substitution impacts negatively on inflation. The impact is highly significant at the 1% level. A 1%

increase in the rate of currency substitution is associated with over 3.2% decline in inflation. This suggests that currency substitution helped to reduce inflation in Nigeria within the sample period. Investment also contributes to reduction of inflation (with a lag of one year) as indicated by the negative coefficient of the investment variable, though it is significant at the 10% level. The effects of government expenditure and trade openness on inflation are statistically not significant. Exchange rate depreciation is associated with increase in inflation rate. This is indicated by the negative coefficient on the exchange rate variable which is significant at the 6% level, though the depreciation of exchange rate affects inflation with a lag of one year. This could be attributed to high level of dependence of the country on imports.

The policy options implied by the foregoing result are that reduction in the rate of inflation in the country would require acceptance of some level of currency substitution, increase in the level of investment and prevention of gross depreciation of the currency.

7. **Conclusion**

The paper attempted to investigate the interrelationships among currency substitution, inflation and economic growth in Nigeria using the two-stage least squares (2SLS) estimation technique to estimate a system of simultaneous equation specified for the purpose. The analysis reveals that inflation and the growth of real GDP have no significant effects on currency substitution, but that exchange rate and investment significantly affect currency substitution in the country. While exchange rate depreciation is associated with increase in the rate of currency substitution, increase in investment reduces it. Inflation was observed to have positively affected real GDP growth. This was attributed to the fact that inflation was below the threshold inflation rate (identified in the literature on threshold effect of inflation in Nigeria) in most of the period, and was thus not detrimental to the growth of the economy. Currency substitution impacts favourably on the growth of real GDP, just as it also contributes to reduction in inflation in the country. Investment helps reduce inflation, but exchange rate depreciation increases it.

REFERENCES

Adom, Assande Desire, Subhash. C. Sharma and AKM Mahbub, Morshed. 2009. "Currency Substitution in Selected African Countries", Journal of Economic Studies, 36(6): pp. 616-640.

Asel, Isakova. 2009. *Currency Substitution in the Economies of Central Asia: How Much Does It Cost?* Paper presented at the 2009 Conference on European Economic Integration, Viena, November.

Bawa, Sani and Ismaila S. Abdullahi, 2012. "Threshold Effect of Inflation on Economic Growth in Nigeria", CBN Journal of Applied Statistics, 3(1): pp.43-63.

Central Bank of Nigeria 2013. *Statistical Bulletin*, CBN, Abuja.

Danladi, Jonathan Datsu. 2013. "Inflation and Sustainable Output Performance in the West African Sub-Region: The Threshold Effect". American Journal of Economics, 3(6): pp. 252-259.

Doguwa, Sani I. 2013. "Inflation and Economic Growth in Nigeria: Detecting the Threshold Level". CBN Journal of Applied Statistics, 3(2): pp. 99-124.

Doguwa, Sani I. 2014. "Currency Substitution: Evidence from Nigeria". CBN Journal of Applied Statistics, 5(2): pp. 1-23.

Fielding, David and Shorthand, Anja. 2002. *How Does Political Violence Affect Currency Substitution? Evidence from Egypt*. Retrieved from http://www.le.ac.uk/economics/research/RePEc/lec/leecon/econ02-6.pdf?uol_r=d307e306 on April 3, 2015.

Gujarati, Damodar N. and Dawn C. Porter.2009. *Basic Econometrics*, Fifth Edition, NewYork, NY: McGraw-Hill/Irwin.

Handa, Jagdish. 2000. *Monetary Economics*, Second Edition, Routledge, Newyork

Sawada, Yasuyuki and Pan. A. Yotopoulos. 2000. *Currency Substitution, Speculation and Financial Crises: Theory and Empirical Analysis*. Retrieved from http://web.stanford.edu/group/siepr/cgi-bin/siepr/?q=system/files/shared/pubs/papers/pdf/99-5.pdf on April 1, 2015.

Yinusa, Olalekan D. 2009. "Macroeconomic Fluctuation and Deposit Dollarization in Sub-Sahara: Evidence from Panel Data". *MPRA Paper No. 16259*. Retrieved from http://mpra.ub.uni-muenchen.de/16259/1/MPRA_paper_16259.pdf on March 27, 2015.

APPENDIX

The procedure for investigating the identification status of the specified system of simultaneous equations used in this paper is based on procedures outlined in Koutsoyianis, A. (1977). *Theory of Econometrics,* Second Edition, Palgrave, New York.

The Order Condition for Identification

For convenience we recall the system of simultaneous equations:

$$LCS_t = a_0 + a_1LINF_t + a_2LINF_{t-1} + a_3LRGDPG_t + a_4LEXRT_t + a_5LINV_{t-1} + u_{1t} \quad 1^*$$

$$LRGDPG_t = b_0 + b_1LCS_t + b_2LCS_{t-1} + b_3LINF_t + b_4LINV_{t-1} + b_5LGOVEXP_t + b_6LTOPEN_t + u_{2t} \quad 2^*$$

$$LINF_t = c_0 + c_1LCS_t + c_2\,LINV_{t-1} + c_3LGOVEXP_t + c_4LEXRT_{t-1} + c_5LTOPEN_t + u_{3t} \quad 3^*$$

An equation belonging to a system of simultaneous equation is identified if

$$(K\text{-}M) \geq (G - 1) \quad 4^*$$

Where K = total number of variables (endogenous and exogenous) in the system or entire model

M = number of variables (endogenous and exogenous) in any particular equation

G = number of structural equation = number of all endogenous variables in the model

(K-M) measures the total number of variables excluded from a particular equation

If $K - M = G - 1$, then the equation is exactly identified; if $K - M > G - 1$, then it is overidentified; if $K - M > G - 1$, then it is not identified or underidentified. For a system to be identified, all the equations of the system must be identified.

For equation 1*, K = 10, M = 6 and G = 3. Substituting into relation 4, we have $10 - 6 > 3 - 1$, i.e. $4 > 2$. Thus, equation 1* is overidentified.

For equation 2*, K = 10, M = 7 and G = 3. Substituting into relation 4, we have $10 - 7 > 3 - 1$, i.e. $3 > 2$. Thus, equation 2* is overidentified.

For equation 3*, K = 10, M = 6 and G = 3. Substituting into relation 4, we have $10 - 6 > 3 - 1$, i.e. $4 > 2$. Thus, equation 3* is overidentified.

The order condition for identification shows that each equation of the model is overidentified. Thus the entire model is overidentified. However, the order condition is a necessary condition for identification, but it is not sufficient. We therefore proceed to testing the identification status of the model using the rank condition.

The Rank Condition for Identification

"The rank condition states that : in a system of G equations, any particular equation is identified if and only if it is possible to construct at least one non-zero determinant of order (G-1) from the coefficients of the variables excluded from that particular equation but contained in other equations of the model." (Koutsoyianis, 1977, p.353).

The model may be re-written in the form

$-LCS_t + a_0 + a_1LINF_t + a_2LINF_{t-1} + a_3LRGDPG_t + a_4LEXRT_t + a_5LINV_{t-1} + u_{1t} = 0$

$-LRGDPG_t+b_0+b_1LCS_t+b_2LCS_{t-1} + b_3LINF_t + b_4LINV_{t-1} + b_5LGOVEXP_t + b_6LTOPEN_t + u_{2t} = 0$

$-LINF_t + c_0 + c_1LCS_t + c_2\,LINV_{t-1} + c_3LGOVEXP_t + c_4LEXRT_{t-1} + c_5LTOPEN_t + u_{3t} = 0$

Ignoring the intercepts and the random disturbances, the table of parameters of the model is as follows.

Equations	Variables (in logs)									
	CS	INF	RGDPG	EXRT	GOVEXP	TOPEN	INF(-1)	INV(-1)	CS(-1)	EXRT(-1)
1st Equation	-1	a_1	a_3	a_4	0	0	a_2	a_5	0	0
2nd Equation	b_1	b_3	-1	0	b_5	b_6	0	b_4	b_2	0
3rd Equation	c_1	-1	0	0	c_3	c_5	0	c_2	0	c_4

To examine the identifiability of the first equation, we strike out the row of coefficients of this equation and then strike out the columns in which non-zero coefficient of the equation appears. This yields

b_5	b_6	b_2	0
c_3	c_5	0	c_4

Clearly we can obtain several non-zero determinants of order 2 (i.e 3-1, where G represents the number of equations) from variables excluded from this equation but included in other equations. Thus the equation is identified.

We repeat the procedure for the other two equations. For the second equation, we have

a_4	a_2	0
0	0	c_4

From this we can obtain several non-zero determinants of order 2 from variables excluded from this equation but included in other equations. Thus the equation is identified.

Lastly, repeating the procedure for the third equation, we have

a_3	a_4	a_2	0
-1	0	0	b_2

Again we observe that several non-zero determinants of order 2 can be obtained from variables excluded from this equation but included in other equations. Thus the equation is identified.

Since all the equations are identified, then the entire model is identified and can therefore be estimated.