The Empirical Econometrics and Quantitative Economics Letters
ISSN 2286 – 7147
Volume 3, Number 1 (March 2014), pp. 11 - 24.

Sales forecasting with limited information: Comparison between cumulative observations and rolling windows

Orakanya Kanjanatarakul [1], Komsan Suriya [2],
Cheng Hsiao [3] and Christian Gourieroux [4]

[1]*Faculty of Management Science, Chiang Mai Rajabhat University*

[2]*Faculty of Economics, Chiang Mai University*
Email: suriyakomsan@gmail.com

[3]*University of Southern California*

[4]*CREST, University of Paris and University of Totonto*

ABSTRACT

This analysis will forecast the sales of an innovative agro-industrial product, feta cheese produced by the Royal Project Foundation in Thailand. It will estimate a logistic model suggested by Stoneman (2010) that presents the S-curve following the product life cycle theory. By the first method, it will add a number of observations to the previous set of observations, which is called the process of cumulative observations. This process includes the cumulative knowledge from the past. By the second method, it will estimate the model by using the rolling windows method which deletes the oldest data after adding the newest ones. The analysis will compare the accuracy of both approaches in terms of mean absolute percentage error. The results show that the method of rolling windows outperforms the cumulative observations. The optimal window width is 15 months.

Keywords: Sales forecasts, innovation, product life cycle, cumulative observations, rolling windows,

JEL Classification: C53, O31, M31

1. Introduction. Agro-industry produces significantly positive impact on nationwide economy of Thailand (Kanjanatarakul and Suriya, 2012). By this impact, the government supports the agro-industry especially via the Royal Project Foundation which extremely empowers the agency and the whole sector (Kanjanatarakul and Suriya, 2013 a).

Sales forecasting of innovative agro-industrial product with limited information is a challenge for both scholars and practitioners. Traditional econometrics usually uses more than 30 observations to construct a time trend. However, practitioners cannot wait for 30 months to have the complete information for such a forecast. In this paper, we propose a method to use limited information from only 3 observations of the sales for the forecasting. The forecasts are adjusted according to the product life cycle theory, which suggests an S-shape evolution of sales over time that can be modeled using the logistic function as suggested by Stoneman (2010).

Kanjanatarakul and Suriya (2013 b) propose the method of least squares with quadratic interpolation to estimate the S-curve. However, a drawback of their work is on the fixed parameter over time. This is because they use the method of cumulative observations which adds another new observation into the model but not delete any observation away. It assumes that from the beginning until the time that is evaluated by the model, the parameters of the model remain the same. This assumption may not be true especially in the long-run.

A possible way to prevent this drawback is to use the method of rolling windows. The method will add a new observation into the model while delete the oldest observation off the model. It will keep the number of observations in the model constant with the updated information. Therefore, the parameters of the model are refreshed.

This study will find out which method between cumulative observations and rolling windows is better. It will compare the results using mean absolute percentage error (MAPE) derived from the out-of-sample test. Moreover, it will figure out the optimal size of the windows if the technique sounds promising. The results of the study will enhance practitioners in agro-industry to accurately forecast the sales of their new products.

2. Literature review. Sales forecasting has its long history. Bass (1969) pioneers the quantitative method of the forecasting. His model is called Bass model. It captures the diffusion of the innovative products in two channels, the innovation and the imitation. This means that two reasons behind the diffusion are, first, people use it because they want to be different from other people and, second, people use it because they want to imitate other people. Bass model is popular among marketers until nowadays.

Stoneman (2010) launches his book on soft innovation. In the book, he proposes another functional form for the sales forecasting. He simply uses the logistic function. This is challenging to Bass model. It does not capture the two dimensions of diffusion which are innovation and imitation like presented by Bass model. However, this simpler functional form may yield more accurate forecasting results.

Kanjanatarakul and Suriya (2012) prove this curiousness by applying both Bass model and logistic function onto a set of empirical data of an innovative agro-industrial product. They find that the logistic function works better than Bass model. Therefore, this study chooses logistic function to be the functional form of the sales forecasts.

The series of works of Kanjanatarakul and Suriya do not stop there. In 2013, they publish another paper regarding the sales forecasting under limited observations. They find that the sufficiency number of observations is between 7 to 24 observations of monthly data. However, the attacks on their works upon the strong assumption of fixed parameters over time lead them to conduct this study to find out whether their traditional technique of cumulative observations or the method of rolling windows will yield more accurate forecasts.

3. Methodology

3.1 Logistic function

Stoneman (2010) suggests scholars to use the logistic function for forecasting sales of new products as follows:

$$V_T = \frac{M}{1+\mathrm{A}*\exp(-\beta T)} \qquad (1)$$

where V_T = Sales of innovative agro-industrial product;
M = Maximum sales of innovative agro-industrial product;
β = Growth parameter;
A = Shift parameter;
T = Time.

3.2 Least squares using quadratic interpolation algorithm

The parameter estimation includes 8 steps as follows:

Step 1: Set three initial values of parameter M. Transform the data by the Logistic transformation into a linear function.

$$ln\left(\frac{V_T/M}{1-V_T/M}\right) - \ln\left(\frac{1}{\mathrm{A}}\right) = \beta\mathrm{T} \qquad (2)$$

Then, estimate parameter β using Ordinary Least Squares (OLS)

Step 2: Take parameter M and β to forecast sales by this formula.

$$V_T = \frac{M}{1 + \mathrm{A} * \exp(-\beta T)}$$

The value of A is calculated by the following formula to fix the y-intercept at the first data of the series (Vo):

$$A = \frac{M}{V_0} - 1 \qquad (3)$$

Step 3: Calculate the Sum Squared Error (SSE).

$$\sum e^2 = \sum_{i=1}^{N}\left(V_T - \widehat{V_T}\right) \qquad (4)$$

Step 4: Calculate the SSE at the three points using the three initial M values.

Step 5: Search for a new M value by Quadratic Interpolation

Step 6: Include the new M with other two previous M values which are located nearest to the new M. Then, estimate parameter β and calculate the SSE again.
Step 7: Repeat steps 5 and 6 for 10,000 iterations.
Step 8: Summarize the values of parameter M and β.

It should be remarked that the estimation of β using OLS with logistic transformation may suffer from the heteroscedasticity problem. The problem may be avoided by using Estimated Generalized Least Squares or EGLS (Judge et al, 1986). However, Kanjanatarakul (2013) shows that OLS is better than EGLS in this case. Therefore, this study uses OLS rather than EGLS in the estimation of β.

4. Data

The data are obtained from the Royal Project Foundation. They are monthly sales of feta cheese. The data cover the period during January 2010 to August 2012. Totally, the model has 32 observations.

5. Results

We will first present the results from the method of rolling windows. The width of the windows was varied from 3 to 19. Due to the large size of the results, they are presented in the appendix section of this chapter. However, two of the best models with width of 15 and 16 observations, which demonstrate the lowest average MAPE in this class, are displayed here.

Table 1: Estimation results for the Logistic function using OLS and quadratic interpolation with the method of rolling windows when the width of window is 15

Repeat	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
1	1,156,423	0.0527	6,000	47.22	19.94	19.99
2	1,482,125	0.0136	17,700	32.89	21.02	21.07
3	1,806,054	0.0213	11,800	34.63	20.61	20.66
4	2,205,003	0.0182	11,100	30.35	20.56	20.61
5	2,729,756	0.0093	21,700	18.97	21.23	21.27
6	3,357,948	0.0231	11,600	27.40	20.60	20.65

Source: Calculation using own programming in Matlab.

Table 2: Estimation results for the Logistic function using OLS and quadratic interpolation with the method of rolling windows when the width of window is 16

Repeat	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
1	1,156,741	0.0520	6,250	49.93	19.91	19.96
2	1,483,627	0.0142	17,900	32.13	20.96	21.01
3	1,802,822	0.0162	13,500	28.85	20.68	20.73
4	2,206,022	0.0232	11,300	32.03	20.50	20.55
5	2,731,507	0.0134	20,500	21.81	21.10	21.15

Source: Calculation using own programming in Matlab.

The study will find the best model with rolling windows using t-test. The results are shown in Tables 3.26 and 3.27.

Table 3: Descriptive statistics of MAPE, AIC and BIC from the estimation methods of rolling windows with widths15 and 16

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	MAPE15	32.810	5	10.107	4.520
	MAPE16	32.948	5	10.376	4.640
Pair 2	AIC15	20.673	5	.496	.222
	AIC16	20.629	5	.467	.209
Pair 3	BIC15	20.720	5	.496	.222
	BIC16	20.677	5	.467	.209

Source: Calculation using SPSS version 11.0.

The MAPE of the model with a width of 15 observations is slightly lower than that with 16 observations. The AIC and BIC of the former model are higher than those of the latter model. However, as shown in Table 3.26, these differences are not statistically significant. Therefore, the best model is the one with 15 observations because it uses fewer observations.

Table 4: Comparison of MAPE, AIC and BIC for the estimation methods of rolling windows with widths 15 and 16
Source: Calculation using SPSS version 11.0.

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	MAPE15 MAPE16	-.138	3.611	1.615	-4.622	4.346	-.085	4	.936
Pair 2	AIC15 AIC16	.044	.071	.032	-.044	.131	1.390	4	.237
Pair 3	BIC15 BIC16	.043	.071	.032	-.045	.130	1.355	4	.247

In the next step, this study will compare the performances of the best model from the method of rolling windows to the best model with the method of cumulative observations. The results are shown in Tables 3.28 and 3.29.

Table 5: Estimation results for the Logistic function using OLS and quadratic interpolation

No. of obs	Mstar	Beta	SSE (Million)	MAPE	AIC	BIC
3	1,801,249	0.2028	828	46.77	20.10	19.80
4	1,479,900	0.1343	965	46.49	19.80	19.65
5	1,481,169	0.1248	987	48.10	19.50	19.42
6	1,803,815	0.0980	1,110	49.09	19.37	19.33
7	1,154,894	0.0420	2,057	43.42	19.78	19.78
8	1,154,819	0.0477	2,013	45.38	19.59	19.60
9	1,170,708	0.0657	2,130	47.97	19.50	19.53
10	1,154,446	0.0486	2,501	45.96	19.54	19.57
11	1,152,311	0.0414	2,732	45.29	19.51	19.55

No. of obs	Mstar	Beta	SSE (Million)	MAPE	AIC	BIC
12	1,152,992	0.0432	2,703	47.69	19.40	19.44
13	1,151,351	0.0365	2,988	46.62	19.41	19.45
14	1,147,465	0.0492	5,261	49.36	19.89	19.93
15	1,156,423	0.0527	6,002	50.58	19.94	19.99
16	1,156,741	0.0520	6,247	51.80	19.91	19.96
17	1,153,404	0.0503	6,400	53.22	19.86	19.91
18	1,471,196	0.0431	6,379	53.38	19.80	19.85
19	1,474,993	0.0468	8,795	54.26	20.06	20.11
20	1,159,226	0.0496	11,191	54.90	20.24	20.29
21	1,481,511	0.0480	11,634	55.94	20.23	20.28
22	1,173,916	0.0501	14,895	56.08	20.42	20.47
23	1,253,215	0.0507	17,641	55.86	20.54	20.59
24	1,188,729	0.0502	19,173	56.12	20.58	20.63
25	1,100,433	0.0523	27,163	54.44	20.89	20.94
26	1,805,585	0.0484	26,768	57.26	20.83	20.88
27	1,807,158	0.0477	28,475	57.46	20.85	20.90
28	1,814,071	0.0475	31,563	56.29	20.91	20.96
29	2,227,125	0.0468	34,114	54.64	20.95	21.00
30	1,837,872	0.0474	40,498	47.21	21.09	21.14
31	3,407,578	0.0455	41,516	39.35	21.08	21.13
32	4,157,133	0.0434	41,343	-	21.04	21.09

Source: Calculation using own programming in Matlab.

Table 6: Descriptive statistics of MAPE, AIC and BIC from the estimation methods of rolling windows with width 15 and the model with the method of cumulative observations (OLS with quadratic interpolation)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	MAPE15	31.909	6	9.306	3.799
	CUMMAPE	46.542	6	2.004	.818
Pair 2	AIC15	20.661	6	.445	.181
	CUMAIC	19.690	6	.260	.106
Pair 3	BIC15	20.708	6	.445	.181
	CUMBIC	19.597	6	.190	.077

Source: Calculation using SPSS version 11.0

The results show that the MAPE of the model with rolling windows is much lower than that of the model with cumulative observations. However, the AIC and BIC of the model with rolling windows are higher than those of the model with cumulative observations. These results are clearly significant at the confidence level of 99%.

Table 7: Comparison of MAPE, AIC and BIC between the estimation methods of rolling window with width 15 and the model with the method of cumulative observations (OLS with quadratic interpolation)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	MAPE15 CUMMAPE	-14.633	8.419	3.437	-23.468	-5.798	-4.26	5	.008
Pair 2	AIC15 CUMAIC	.971	.572	.234	.370	1.571	4.15	5	.009
Pair 3	BIC15 CUMBIC	1.111	.478	.195	.609	1.613	5.69	5	.002

Source: Calculation using SPSS version 11.0.

To judge which one is better, the purpose of the prediction should be the first priority. The model with smaller MAPE calculated from the out-of-sample test should be considered better. Therefore, the model using rolling windows should be preferred to the model using cumulative observations.

6. Conclusions and further studies. This study answers the research question whether the method of rolling windows will give more accurate forecasting results than the method of cumulative observations. It will also find out, if the rolling window is better, the optimal size of the windows. It uses a set of empirical data of an innovative agro-industrial product, feta cheese produced by the Royal Project Foundation in Thailand to prove this curiousness. It measures the accuracy of the models by mean absolute percentage error (MAPE).

It is clear from the results that the method of rolling windows outperforms the method of cumulative observations. The optimal window width is 15 months. This confirms that the assumption of floating parameters over time is more suitable than the fixed parameters over time. It is because the rolling windows can adaptively capture the change of the sales situation which evolves continuously along the product life cycle.

The challenge of forecasting with limited information is still going on. Even though the big data becomes more and more important and may threaten social research dramatically (Suriya, 2013) the limited data cannot be ignored by social researchers. A further technique on sales forecast of innovative agro-industrial product is the forecasting using belief functions. This is the development of Kanjanatarakul, Sriboonchitta and Denoeux (2014).

Ultimately, we hope that these developments of the use of limited information to accurately forecast the sales or to analyze them in efficient and creative ways will enhance economists to work on limited resources that may not be avoided in conducting field research. The study should not be restricted to only innovative agro-industrial products (Kanjanatarakul and Suriya, 2013 c). The techniques should be extended to other fields such as innovative tourism products (Suriya, 2012; Srichoochart and Suriya, 2012) and new transportation services (Suriya et al, 2012) that practitioners usually collect only limited data of their sales in the introductory stage and would like to know whether the new products or services should be continuously supported or terminated. This is crucial for business to make the decision.

References

Bass, Frank. 1969. "A New Product Growth Model for Consumer Durables," Management Science 15 (January): pp. 215 – 227.

Judge, George, et al. 1986. Introduction to the Theory and Practice of Econometrics. 2nd ed. New York: John Wiley and Sons.

Kanjanatarakul, Orakanya, 2013. An Economic Analysis of the Royal Project Foundation: Sales Forecast of Feta Cheese and Impact on Thai Economy. Doctoral dissertation, Faculty of Economics, Chiang Mai University.

Kanjanatarakul, Orakanya and Komsan Suriya. 2012. "Economic impact of agro-industrial sector on nationwide economy of Thailand: A general equilibrium approach," The Empirical Econometrics and Quantitative Economics Letters 1, 4 (December 2012): pp. 61 – 66.

Kanjanatarakul, Orakanya and Komsan Suriya. 2012. "Comparison of sales forecasting models for an innovative agro-industrial product: Bass model versus logistic function," The Empirical Econometrics and Quantitative Economics Letters 1, 4 (December 2012): pp. 89 – 106.

Kanjanatarakul, Orakanya and Komsan Suriya. 2013 a. "Role of government funding on the empowerment of the Royal Project Foundation: A general equilibrium approach," The Empirical Econometrics and Quantitative Economics Letters 2, 1 (March 2013): pp. 1 - 28.

Kanjanatarakul, Orakanya and Komsan Suriya. 2013 b. "Sufficient Number of Observations for Sales Forecasts with Limited Information: A Case of an Innovative Agro-Industrial Product," International Journal of Intelligent Technologies and Applied Statistics 6, 2 (June 2013), pp. 121 - 134.

Kanjanatarakul, Orakanya and Komsan Suriya. 2013 c. "Forecasting the sales of an innovative agro-industrial product with limited information: A case of feta cheese from buffalo milk in Thailand," Proceedings in the Ecomod2013 conference in Prague.

Kanjanatarakul, Orakanya, Songsak Sriboonchitta and Thierry Denoeux. 2014. "Forecasting using belief functions: An application to marketing econometrics," International Journal of Approximate Reasoning. In press (February 2014).

Stoneman, Paul. 2010. Soft Innovation: Economics, Product Aesthetics and the Creative Industries. Oxford: Oxford University Press.

Srichoochart, Siriporn and Komsan Suriya. 2012. "Factors influencing the intention to purchase an innovative tourism service for learning Northern Thai intellectualities using a conditional Logit model," In Peter Calkins et al (eds.). 2012. Unifying Econometrics: Bridging the Gap between Theory and Practice. Chiang Mai: CMSE Press, pp. 233-238.

Suriya, Komsan. 2013. "Big data," The Empirical Econometrics and Quantitative Economics Letters 2, 4 (December 2013): pp. 87 – 88. (Book review)

Suriya, Komsan. 2012. "An economic analysis of tourism demand and innovative tourism products: A case study of Lampang province in Thailand," The Empirical Econometrics and Quantitative Economics Letters 1, 1 (March 2012): pp. 49 – 68.

Suriya, Komsan et al. 2012. "New services development and pricing strategy of rail transporter in Thailand," The Empirical Econometrics and Quantitative Economics Letters 1, 1 (March 2012): pp. 39 – 48.

Annex

The estimation results of the Logistic function using rolling windows are presented here, with the width of window ranges from 3 to 20.

Table A-1 The estimation results of the Logistic function using OLS and quadratic interpolation with the method of rolling window when the width of window is 3.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,801,249	0.2028	827,752,213	28.60	20.10	19.80
2	2,196,933	-0.0336	5,608,479,871	130.40	22.02	21.72
3	2,694,513	0.0183	3,135,837,752	71.58	21.43	21.13
4	3,332,660	0.0185	2,686,089,049	70.01	21.28	20.98
5	4,103,375	-0.0651	13,774,806,696	202.12	22.91	22.61
6	5,073,402	-0.0279	7,186,289,977	92.02	22.26	21.96
7	6,264,092	0.0894	1,118,112,985	43.34	20.40	20.10
8	7,707,403	0.0011	4,522,025,036	59.72	21.80	21.50
9	9,496,986	-0.0532	41,831,558,814	255.13	24.03	23.72
10	11,747,851	0.0224	876,007,444	38.01	20.16	19.86
11	14,465,776	0.0080	2,231,801,379	30.49	21.09	20.79
12	17,834,933	0.0106	4,601,992,414	40.01	21.82	21.52
13	22,001,390	0.0481	2,551,572,578	40.03	21.23	20.93
14	27,112,342	-0.0205	60,623,759,035	244.80	24.40	24.10
15	33,446,352	-0.0103	22,564,367,498	114.82	23.41	23.11
16	41,267,417	-0.0139	18,288,384,911	61.21	23.20	22.90
17	50,901,314	0.0004	8,218,213,154	33.52	22.40	22.10
18	62,807,207	0.0391	3,530,561,467	44.85	21.55	21.25

Source: Calculation using Matlab version 2007.

Table A-2 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 4.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,479,900	0.1343	9.65E+08	30.23	19.80	19.65
2	1,814,234	-0.006	6.27E+09	100.92	21.67	21.52
3	2,220,276	0.0117	4.3E+09	77.14	21.30	21.14
4	2,752,891	-0.0277	7.1E+09	89.90	21.80	21.64
5	3,387,220	-0.0432	1.47E+10	144.56	22.53	22.37
6	4,167,552	0.005	5.25E+09	62.05	21.49	21.34
7	5,138,693	0.0703	1.05E+09	45.71	19.88	19.73
8	6,329,590	-0.0059	7.43E+09	51.73	21.84	21.69
9	7,800,831	-0.0444	4.92E+10	217.54	23.73	23.58
10	9,627,524	0.0179	1.36E+09	36.86	20.14	19.99
11	11,854,787	0.0303	2.62E+09	31.08	20.80	20.65
12	14,625,119	0.0158	4.43E+09	36.13	21.33	21.17
13	18,035,085	0.0448	2.54E+09	42.65	20.77	20.62
14	22,220,133	-0.025	9.77E+10	255.14	24.42	24.26
15	27,422,701	-0.0209	4.7E+10	116.14	23.69	23.53
16	33,821,129	-0.0022	1.39E+10	42.30	22.47	22.31
17	41,726,283	0.008	6.73E+09	29.86	21.74	21.59

Source: Calculation using Matlab version 2007.

Table A-3 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 5.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,481,169	0.1248	9.87E+08	34.00	19.50	19.42
2	1,853,969	-0.0087	8.11E+09	108.31	21.61	21.53
3	2,240,391	-0.0315	9.3E+09	98.12	21.74	21.67
4	2,786,427	-0.0116	7.31E+09	67.57	21.50	21.43
5	3,414,637	-0.0134	1.19E+10	96.52	21.99	21.92
6	4,206,172	-0.0065	8.27E+09	57.35	21.63	21.55
7	5,185,882	0.0632	1.02E+09	49.73	19.54	19.46
8	6,390,211	-0.0008	8.05E+09	43.84	21.60	21.52
9	7,870,546	-0.0475	6.99E+10	201.90	23.76	23.68
10	9,697,855	0.0379	2.93E+09	40.32	20.59	20.51
11	11,962,451	0.0357	2.76E+09	34.17	20.53	20.45
12	14,750,244	0.0149	5.24E+09	33.57	21.17	21.09
13	18,188,491	0.0414	2.48E+09	45.43	20.42	20.34
14	22,418,866	-0.035	1.75E+11	266.53	24.68	24.60
15	27,664,440	-0.0119	4.11E+10	94.97	23.23	23.15
16	34,122,244	0.0042	1.19E+10	39.70	21.99	21.91

Source: Calculation using Matlab version 2007.

Table A-4 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 6.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,803,815	0.0980	1.11E+09	34.07	19.37	19.33
2	2,211,780	-0.0486	1.53E+10	142.14	21.99	21.96
3	2,746,142	-0.0158	9.66E+09	73.46	21.53	21.50
4	3,373,283	0.0137	5.84E+09	52.27	21.03	21.00
5	4,152,890	-0.0230	1.76E+10	94.43	22.13	22.10
6	5,120,336	-0.0082	1.05E+10	47.93	21.61	21.58
7	6,309,057	0.0655	1.25E+09	53.00	19.49	19.45
8	7,777,619	-0.0059	1.15E+10	35.07	21.70	21.67
9	9,579,744	-0.0278	5.43E+10	156.04	23.26	23.22
10	11,815,605	0.0435	3.47E+09	43.17	20.51	20.47
11	14,570,887	0.0352	2.77E+09	37.75	20.28	20.25
12	17,967,683	0.0134	6.35E+09	29.84	21.11	21.08
13	22,155,757	0.0318	2.61E+09	42.70	20.22	20.19
14	27,328,575	-0.0278	1.71E+11	238.67	24.41	24.37
15	33,712,429	-0.0065	3.87E+10	89.64	22.92	22.89

Source: Calculation using Matlab version 2007.

Table A-5 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 7.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,154,894	0.0420	2.06E+09	33.04	19.78	19.78
2	1,493,792	-0.0312	1.53E+10	102.68	21.79	21.78
3	1,816,219	0.0096	7.67E+09	55.85	21.10	21.09
4	2,221,951	0.0023	8.13E+09	48.16	21.16	21.15
5	2,750,891	-0.0238	2.12E+10	81.04	22.12	22.11
6	3,382,662	-0.0015	1.03E+10	39.71	21.39	21.39
7	4,162,850	0.0582	1.2E+09	58.00	19.25	19.24
8	5,123,681	0.0103	8.8E+09	33.94	21.24	21.23
9	6,313,560	-0.0201	5.24E+10	132.89	23.02	23.01
10	7,785,850	0.0437	3.61E+09	46.36	20.35	20.34
11	9,597,848	0.0339	2.76E+09	41.27	20.08	20.07
12	11,831,621	0.0058	1.08E+10	17.74	21.44	21.43
13	14,588,867	0.0369	4.23E+09	44.56	20.50	20.50
14	18,003,821	-0.0230	1.71E+11	231.81	24.21	24.20

Source: Calculation using Matlab version 2007.

Table A-6 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 8.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,154,819	0.0477	2.01E+09	36.34	19.59	19.60
2	1,485,975	-0.0040	1.28E+10	73.23	21.44	21.45
3	1,809,840	-0.0014	1.04E+10	51.75	21.24	21.24
4	2,214,369	-0.0005	9.78E+09	41.66	21.17	21.18
5	2,744,569	-0.0164	2.15E+10	64.02	21.96	21.97
6	3,372,630	-0.0046	1.27E+10	32.15	21.44	21.45
7	4,134,633	0.0709	5.19E+09	58.76	20.54	20.55
8	5,105,789	0.0159	8.12E+09	33.32	20.99	21.00
9	6,287,638	-0.0174	5.69E+10	119.10	22.94	22.95
10	7,754,588	0.0426	3.68E+09	49.37	20.20	20.21
11	9,554,299	0.0265	3.33E+09	37.26	20.10	20.11
12	11,781,719	0.0115	9.34E+09	18.55	21.13	21.14
13	14,532,486	0.0398	5.89E+09	44.05	20.67	20.68

Source: Calculation using Matlab version 2007.

Table A-7 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 9.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,170,708	0.0657	2.13E+09	37.41	19.50	19.53
2	1,493,980	-0.0137	1.67E+10	66.12	21.56	21.59
3	1,816,533	-0.0038	1.23E+10	44.11	21.26	21.28
4	2,226,606	0.0050	9.69E+09	36.47	21.02	21.04
5	2,757,631	-0.0185	2.61E+10	51.82	22.01	22.03
6	3,376,734	0.0114	1.05E+10	31.33	21.10	21.12
7	4,157,326	0.0736	6.75E+09	60.12	20.66	20.68
8	5,128,033	0.0169	8.26E+09	33.12	20.86	20.88
9	6,317,481	-0.0160	6.28E+10	108.17	22.89	22.91
10	7,783,130	0.0357	3.71E+09	49.72	20.06	20.08
11	9,597,384	0.0314	4.47E+09	39.28	20.25	20.27
12	11,835,431	0.0153	8.58E+09	20.95	20.90	20.92

Source: Calculation using Matlab version 2007.

Table A-8 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 10.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,154,446	0.0486	2.5E+09	37.81	19.54	19.57
2	1,484,319	-0.0150	1.94E+10	55.11	21.59	21.62
3	1,810,290	0.0019	1.22E+10	37.84	21.12	21.15
4	2,214,679	0.0014	1.16E+10	30.57	21.07	21.11
5	2,729,501	-0.0020	2.08E+10	41.23	21.65	21.69
6	3,357,911	0.0175	9.79E+09	32.04	20.90	20.93
7	4,135,158	0.0721	7.44E+09	61.92	20.63	20.66
8	5,098,830	0.0168	8.61E+09	32.22	20.77	20.80
9	6,279,733	-0.0203	8.14E+10	93.17	23.02	23.05
10	7,739,146	0.0403	5.79E+09	51.08	20.38	20.41
11	9,547,785	0.0345	5.75E+09	38.88	20.37	20.40

Source: Calculation using Matlab version 2007.

Table A-9 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 11.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,152,311	0.0414	2.73E+09	39.19	19.51	19.55
2	1,484,715	-0.0083	1.94E+10	45.09	21.47	21.51
3	1,809,364	-0.0014	1.44E+10	30.96	21.17	21.21
4	2,202,041	0.0165	1.02E+10	29.75	20.83	20.87
5	2,726,638	0.0048	1.93E+10	36.48	21.46	21.50
6	3,352,558	0.0190	9.72E+09	33.23	20.78	20.82
7	4,130,691	0.0696	7.95E+09	63.68	20.58	20.62
8	5,088,344	0.0112	1.13E+10	22.87	20.93	20.97
9	6,266,672	-0.0136	7.34E+10	80.29	22.80	22.84
10	7,731,124	0.0432	7.89E+09	50.56	20.57	20.61

Source: Calculation using Matlab version 2007.

Table A-10 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 12.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,152,992	0.0432	2.7E+09	43.28	19.40	19.44
2	1,484,451	-0.0109	2.25E+10	35.13	21.52	21.56
3	1,797,008	0.0139	1.25E+10	30.25	20.93	20.97
4	2,200,964	0.0221	9.67E+09	31.44	20.67	20.71
5	2,722,433	0.0071	1.93E+10	32.97	21.37	21.41
6	3,349,217	0.0193	9.84E+09	33.92	20.69	20.73
7	4,120,717	0.0619	7.64E+09	68.28	20.44	20.48
8	5,080,148	0.0164	1.05E+10	24.38	20.76	20.80
9	6,255,391	-0.0087	6.82E+10	76.75	22.63	22.67

Source: Calculation using Matlab version 2007.

Table A-11 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 13.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,151,351	0.0365	2.99E+09	44.75	19.41	19.45
2	1,470,407	0.0052	1.9E+10	34.07	21.26	21.30
3	1,794,725	0.0197	1.18E+10	31.48	20.78	20.82
4	2,195,132	0.0234	9.59E+09	34.00	20.57	20.62
5	2,717,295	0.0080	1.98E+10	29.58	21.30	21.34
6	3,337,425	0.0142	1.18E+10	27.15	20.78	20.82
7	4,105,223	0.0648	1.13E+10	68.58	20.74	20.78
8	5,067,061	0.0200	1.03E+10	25.00	20.64	20.68

Source: Calculation using Matlab version 2007.

Table A-12 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 14.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,147,465	0.0492	5.26E+09	45.26	19.89	19.93
2	1,473,992	0.0116	1.78E+10	33.15	21.11	21.15
3	1,798,845	0.0211	1.17E+10	33.31	20.68	20.73
4	2,199,800	0.0235	9.64E+09	35.89	20.49	20.54
5	2,720,083	0.0036	2.37E+10	18.43	21.39	21.44
6	3,343,677	0.0195	1.15E+10	28.49	20.67	20.71
7	4,113,136	0.0664	1.49E+10	67.85	20.93	20.97

Source: Calculation using Matlab version 2007.

Table A-13 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 15.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,156,423	0.0527	6E+09	47.22	19.94	19.99
2	1,482,125	0.0136	1.77E+10	32.89	21.02	21.07
3	1,806,054	0.0213	1.18E+10	34.63	20.61	20.66
4	2,205,003	0.0182	1.11E+10	30.35	20.56	20.61

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
5	2,729,756	0.0093	2.17E+10	18.97	21.23	21.27
6	3,357,948	0.0231	1.16E+10	27.40	20.60	20.65

Source: Calculation using Matlab version 2007.

Table A-14 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 16.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,156,741	0.0520	6.25E+09	49.93	19.91	19.96
2	1,483,627	0.0142	1.79E+10	32.13	20.96	21.01
3	1,802,822	0.0162	1.35E+10	28.85	20.68	20.73
4	2,206,022	0.0232	1.13E+10	32.03	20.50	20.55
5	2,731,507	0.0134	2.05E+10	21.81	21.10	21.15

Source: Calculation using Matlab version 2007.

Table A-15 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 17.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,153,404	0.0503	6.4E+09	52.61	19.86	19.91
2	1,478,422	0.0095	2.06E+10	23.26	21.03	21.08
3	1,801,119	0.0213	1.34E+10	30.06	20.60	20.65
4	2,206,081	0.0267	1.18E+10	31.32	20.47	20.52

Source: Calculation using Matlab version 2007.

Table A-16 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 18.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,471,196	0.0431	6.38E+09	54.62	19.80	19.85
2	1,795,268	0.0149	1.99E+10	24.64	20.93	20.98
3	2,197,795	0.0247	1.38E+10	29.18	20.57	20.62

Source: Calculation using Matlab version 2007.

Table A-17 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 19.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,474,993	0.0468	8.8E+09	55.45	20.06	20.11
2	1,801,382	0.0188	1.94E+10	25.23	20.85	20.90

Source: Calculation using Matlab version 2007.

Table A-18 The estimation results of the Logistic Model using OLS and quadratic interpolation with the method of rolling window when the width of window is 20.

Repeat	Mstar	beta	SSE	MAPE	AIC	BIC
1	1,159,226	0.05	1.12E+10	54.90	20.24	20.29

Source: Calculation using Matlab version 2007.