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Sustainability of profit and corporate social responsibility Mathematical modeling with phase diagram

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ABSTRACT

Business in modern world targets both profit and social responsibility. In the trading-off between making profits and spending for social responsibility, a firm needs to balance these two goals and maintain them in the long-run. Sustainable profit will prevent the firm to fall into the decaying stage of the product life cycle. Sustainable social responsibility will ensure that the business consistently contributes social benefits. This study develops a mathematical model using phase diagram to find a steady state and stable path for a firm to achieve the sustainability of both profit and social responsibility at the same time. It indicates four areas that are crucial to the operation and performance of the firm. They consist of warm glow area, frozen area, charitable area and decayed area. Major results indicate that a firm cannot automatically achieve both sustainable profit and social responsibility at the same time. It needs policy manipulation and well-planned adjustment to move the business to the twin sustainability. Without manipulation, a firm located in warm glow area is expected to achieve only sustainable profit but it cannot easily stop spending for social responsibility. A firm located in frozen area can achieve only sustainable profit too but it has to shrink its social responsibility. The society will enjoy a firm located in charitable area when it spends too much for social responsibility and face the risk of bankruptcy. When the firm is located in the decayed area, it will face the downfalls of both the profit and expenditure for social responsibility.

Keywords: Theory of the firm, mathematical modeling, phase diagram, sustainable development, corporate social responsibility.

JEL Classification: D21, C62, O12

1. Sustainability of profit

A firm operates for profit. However, the profit grows and drops along the product life cycle (Vernon, 1966). Through the product life cycle, first the profit will grow slowly in the introduction stage. Then it rises sharply in the growth stage. The profit is quite stable in the maturity stage. Finally, it declines in the decline stage.

A dream of avoiding the decline can be dated back to 220 – 120 BC in the era of Qin Shi Huang, the First Emperor of China. In his last years, he tried hard to seek for immortality. He sent explorers to every corner of the world to find it. However, he could not find the pill. The conquest for immortality was not achieved then.

The idea of immortality is translated into the sustainability in the modern world. Many study aims at sustainability of the industry such as in tourism sector (Suriya, 2012; Suriya and Gruen, 2012) and organic agriculture (Sudtasan and Suriya, 2012). A firm or a business seems unable to avoid the decline stage of its profit. However, to dream this sustainability, a firm may find a steady state that yields the long-run stable profit without decline. The idea is against Vernon's life cycle theory such that the profit will not decline in the end but keep stable for a long time (Figure 1).

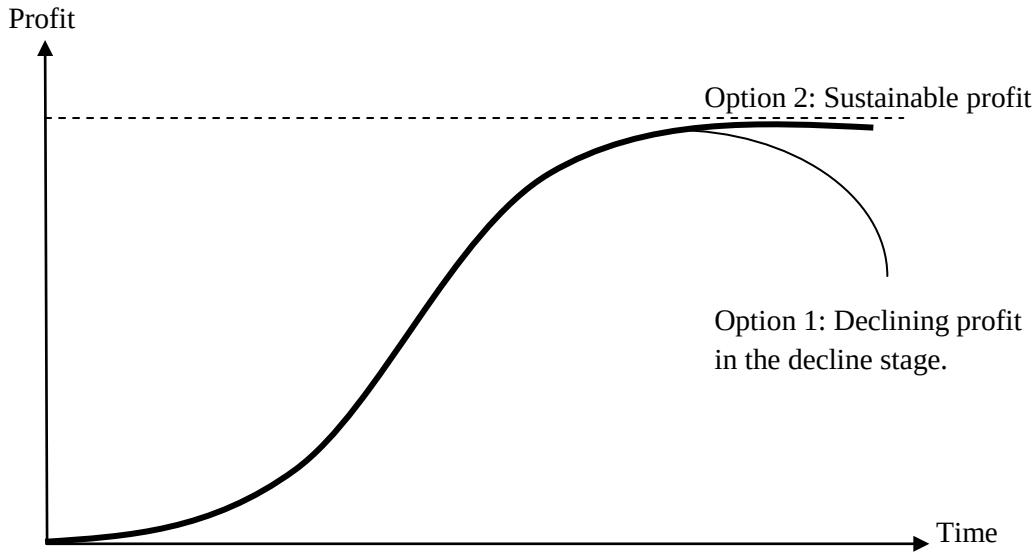

Figure 1: Product life cycle with two options of declining profit in the decline stage and sustainable profit

Maximized profit is viewed as the ultimate goal of a firm in traditional economics. However, in the modern world, business needs to satisfy many public interests. The analysis needs to deal with optimization of multiple targets. Among the external targets, multiple dimension of poverty is at a major concern of society (Suriya, 2008). Pollutions and hazard risks are also set as targets in many industries especially the energy sector (Sudtasan and Suriya, 2014).

2. Sustainability of corporate social responsibility

In the modern world, business cannot deny social responsibility. The idea of social responsibility of business can be dated back to1970 by Milton Friedman. He said that the social responsibility was not only good for society but also the firm itself by the increasing profit. The expenditure spent for social responsibility increases the popularity of the firm with customers. Then they tend to support the firm by increasing the amount of purchase and lead the firm's profit to increase. This idea is called Corporate Social Responsibility (CSR) nowadays. It turns to be a commitment of business and an expectation for sustainable business (D'Amato, Henderson and Florence, 2009) and corporate competitiveness (European Commission, 2008).

In a macroeconomic perspective, corporate social responsibility strengthens the economy by the income distribution from share holders of the business to people outside the business. As stated by Techanan and Suriya (2012), income distribution can help reducing poverty too.

However, the expenditure for social responsibility is accounted as a cost of the firm. Raising the expenditure too much may harm the profit. When the business is out of control of the financial stability, both the profit and corporate social responsibility fluctuate. This instability weakens both the benefits of the firm and society.

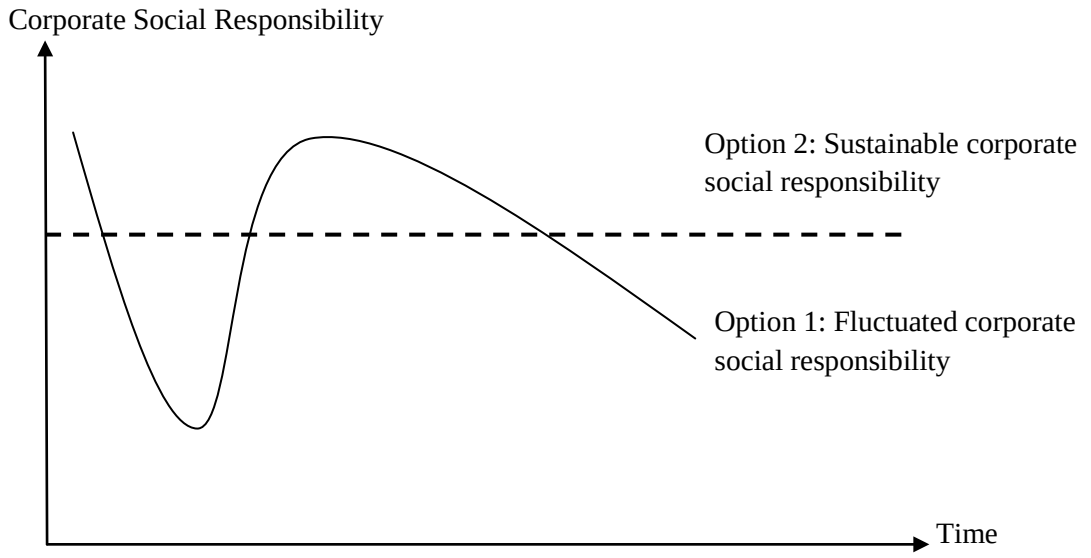

Figure 2: Corporate social responsibility with two options of fluctuated and stable expenditure

A sustainable corporate social responsibility is preferable to the fluctuated one (Figure 2). It will ensure that the society receive the merits from business constantly in the long term. Even though the amount of the aid may not much but the consistence may be perceived as a better way to provide the aid (WaterAid, 2011).

3. Model

This section presents the mathematical modeling of sustainable profit and corporate social responsibility. First, it will describe the settings of the profit function. Second, it will find the sustainability of profit. Third, it will show the sustainability of corporate social responsibility. Last, it will construct a phase diagram and locate the steady state with stable paths to the equilibrium.

3.1 Settings of the profit function

The profit of a firm is the difference between total revenue (TR) and total cost (TC).

$$\pi = TR - TC$$

Total revenue comes from the product of price (P) and quantity sold to the market (Q). Total cost consists of fixed cost (FX), maintenance cost (δFX) when δ is the depreciation rate, variable cost (CQ) when C is the unit cost, the research and development expenditure (RD) and expenditure for the corporate social responsibility (S).

$$\pi = PQ - [FX + \delta FX + CQ + RD + S]$$

Rearrange the term,

$$\pi = (P - C)Q - [(1 + \delta)FX + RD + S]$$

Define Φ as a unit profit, $(P - C) = \Phi$.

$$\pi = \Phi Q - [(1 + \delta)FX + RD + S]$$

Profit change over time can be derived from the first derivative of profit over time.

$$\frac{d\pi}{dt} = \Phi \frac{dQ}{dt} + Q \frac{d\Phi}{dt} - (1 + \delta) \frac{dFX}{dt} - \frac{dRD}{dt} - \frac{dS}{dt}$$

Define the effect of corporate social responsibility on the quantity sold to the market as a logarithm function of corporate social responsibility to the power of α (Figure 3). This function displays two important properties. First, the effect is diminishing. Second, there exists a reservation value before the corporate social responsibility translates into the effect.

$$Q = (lnS)^{\alpha} \text{ and } S = e^{lnS}$$

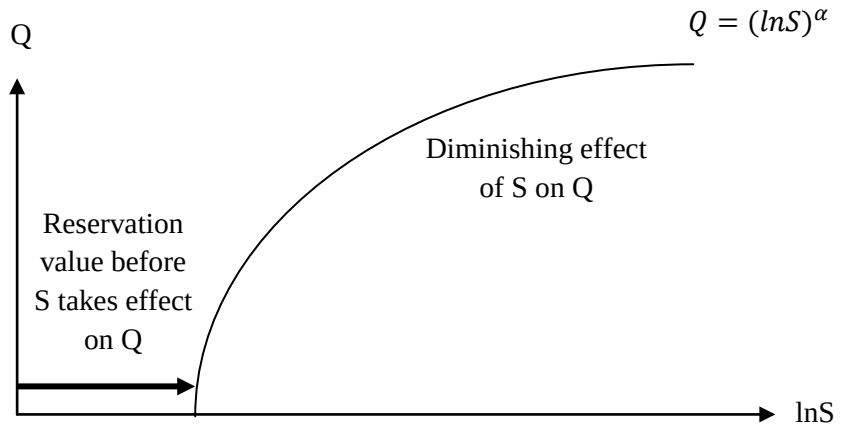

Figure 3: Effect of corporate social responsibility on quantity sold to the market

Translate the profit function according to the definitions above and find the derivatives on the right-hand side (RHS).

$$\frac{d\pi}{dt} = \Phi \frac{dQ}{dlnS}\frac{dlnS}{dt} + Q\frac{d\Phi}{dt} - (1+\delta)\frac{dFX}{dt} - \frac{dRD}{dt} - \frac{de^{lnS}}{dlnS}\frac{dlnS}{dt}$$

$$\frac{d\pi}{dt} = \Phi\alpha(lnS)^{\alpha-1}\frac{dlnS}{dt} + (lnS)^{\alpha}\frac{d\Phi}{dt} - (1+\delta)\frac{dFX}{dt} - \frac{dRD}{dt} - e^{lnS}\frac{dlnS}{dt}$$

Change the symbol of the derivative over time as the dot. This is the change of profit over time.

$$\dot{\boldsymbol{\pi}} = \Phi\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}} + (lnS)^{\alpha}\dot{\boldsymbol{\Phi}} - (1+\delta)\dot{\boldsymbol{FX}} - \dot{\boldsymbol{RD}} - e^{lnS}\dot{\boldsymbol{lnS}} \qquad (1)$$

3.2 Sustainability of profit

The condition of sustainable profit is the change of profit over time is zero.

$$\dot{\boldsymbol{\pi}} = 0$$

Rearrange (1) when the left-hand side (LHS) equals to zero. This yields function (2).

$$\Phi = \frac{(1+\delta)\dot{\boldsymbol{FX}} + \dot{\boldsymbol{RD}} + e^{lnS}\dot{\boldsymbol{lnS}} - (lnS)^{\alpha}\dot{\boldsymbol{\Phi}}}{\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}}$$

Evaluate the value at vertical axis.

$$\lim_{lnS\to 0} \Phi = \frac{(1+\delta)\dot{\boldsymbol{FX}} + \dot{\boldsymbol{RD}} + \dot{\boldsymbol{lnS}}}{0} = \infty$$

It means that when lnS approaches zero, then the unit price approaches infinity.
Take the first derivative of (2) to find the slope of the function. Assume $\dot{\boldsymbol{lnS}}$,$\dot{\boldsymbol{\Phi}}$, $\dot{\boldsymbol{FX}}$, $\dot{\boldsymbol{RD}}$ are constant and positive.

$$\frac{\partial \Phi}{\partial lnS} = \frac{\begin{array}{c}\left[\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}\left\{e^{lnS}\dot{\boldsymbol{lnS}} - \alpha(lnS)^{\alpha-1}\dot{\boldsymbol{\Phi}}\right\}\right] - \\ \left\{(1+\delta)\dot{\boldsymbol{FX}} + \dot{\boldsymbol{RD}} + e^{lnS}\dot{\boldsymbol{lnS}} - (lnS)^{\alpha}\dot{\boldsymbol{\Phi}}\right\}\left\{\alpha(\alpha-1)(lnS)^{\alpha-2}\dot{\boldsymbol{lnS}}\right\}\end{array}}{\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}\ \alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}}$$

$$\frac{\partial \Phi}{\partial lnS} = \frac{\begin{array}{c}\left[\left\{e^{lnS}\dot{\boldsymbol{lnS}} - \alpha(lnS)^{\alpha-1}\dot{\boldsymbol{\Phi}}\right\}\right] - \\ \left\{(1+\delta)\dot{\boldsymbol{FX}} + \dot{\boldsymbol{RD}} + e^{lnS}\dot{\boldsymbol{lnS}} - (lnS)^{\alpha}\dot{\boldsymbol{\Phi}}\right\}\left\{(\alpha-1)(lnS)^{-1}\right\}\end{array}}{\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}}$$

Evaluate at α=1 to simplify the function.

$$\frac{\partial \Phi}{\partial lnS} = \frac{e^{lnS}\dot{\boldsymbol{lnS}} - \dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}$$

This function may have an extremum. Evaluate the zero slope.

$$\frac{\partial \Phi}{\partial lnS} = 0 = \frac{e^{lnS}\dot{\boldsymbol{lnS}} - \dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}$$

Then,

$$0 = e^{lnS}\dot{\boldsymbol{lnS}} - \dot{\boldsymbol{\Phi}}$$

Rearrange the equation.

$$e^{lnS} = \frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}$$

Find lnS instead of e^{lnS} at the extremum to locate the extremum. This is function (3).

$$lnS = \ln\left(\frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}\right)$$

The result shows that there exists an extremum when lnS is positive in (3).

By the LHS of the extremum, the function is decreasing.

$$lnS < \ln\left(\frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}\right) or\ e^{lnS} < \frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}} \Rightarrow \frac{\partial \Phi}{\partial lnS} < 0$$

By the RHS of the extremum, the function is increasing.

$$lnS > \ln\left(\frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}}\right) or\ e^{lnS} > \frac{\dot{\boldsymbol{\Phi}}}{\dot{\boldsymbol{lnS}}} \Rightarrow \frac{\partial \Phi}{\partial lnS} > 0$$

Find the second derivative to ensure the U-shape of the function.

$$\frac{\partial^2 \Phi}{\partial lnS^2} = \frac{1}{\dot{\boldsymbol{lnS}}} elnS\dot{\boldsymbol{lnS}} = elnS > 0$$

Therefore, the positive slope of slope indicates that the function is U-shape and the extremum of the function is the minimum.

3.3 Sustainability of corporate social responsibility

Recall the change of profit over time (1).

$$\dot{\boldsymbol{\pi}} = \Phi\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}} + (lnS)^{\alpha}\dot{\boldsymbol{\Phi}} - (1+\delta)\dot{\boldsymbol{FX}} - \dot{\boldsymbol{RD}} - \dot{\boldsymbol{S}}$$

The condition of sustainable corporate social responsibility (CSR) is the change of CSR over time is zero.

$$\dot{\boldsymbol{S}} = 0$$

This condition affects the change of lnS over time too. Consider the component of the change of lnS over time.

$$\dot{\boldsymbol{S}} = \frac{de^{lnS}}{dlnS}\frac{dlnS}{dt}$$

When $\dot{\boldsymbol{S}} = 0$, then the change of lnS over time is zero too.

$$\frac{dlnS}{dt} = \dot{\boldsymbol{lnS}} = 0$$

This is because the first element of $\dot{\boldsymbol{S}}$ can be zero only if S is zero. However, this is not the case in this study because social responsibility should be positive to ensure that business spends the expenditure for corporate social responsibility.

$$S = \frac{de^{lnS}}{dlnS} \neq 0$$

This result follows the logic that the logarithm of S cannot change when S does not change.

Consider (1) when the change of CSR and logarithm of CSR over time are zero.

$$\dot{\boldsymbol{\pi}} = (lnS)^{\alpha}\dot{\boldsymbol{\Phi}} - (1+\delta)\dot{\boldsymbol{F}}\boldsymbol{X} - \dot{\boldsymbol{R}}\boldsymbol{D}$$

Rearrange the equation. This is function (4).

$$lnS = \left(\frac{\dot{\boldsymbol{\pi}} + (1+\delta)\dot{\boldsymbol{F}}\boldsymbol{X} + \dot{\boldsymbol{R}}\boldsymbol{D}}{\dot{\boldsymbol{\Phi}}}\right)^{\frac{1}{\alpha}}$$

It is clear in (4) that lnS does not depend on unit profit (Φ). Therefore, the graph is a straight line with a 90 degree angle to the horizontal axis.

3.4 Phase diagram

Recall (1).

$$\dot{\boldsymbol{\pi}} = \Phi\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}} + (lnS)^{\alpha}\dot{\boldsymbol{\Phi}} - (1+\delta)\dot{\boldsymbol{F}}\boldsymbol{X} - \dot{\boldsymbol{R}}\boldsymbol{D} - \dot{\boldsymbol{S}}$$

Find the direction of the change of profit over time along with the variety of unit profit on the vertical axis. Assume $\dot{\boldsymbol{lnS}}$,$\dot{\boldsymbol{\Phi}}$, $\dot{\boldsymbol{F}}\boldsymbol{X}$, $\dot{\boldsymbol{R}}\boldsymbol{D}$, $\dot{\boldsymbol{S}}$ are constant and positive.

$$\frac{\partial\dot{\boldsymbol{\pi}}}{\partial\Phi} = \alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}}$$

Evaluate at $\alpha = 1$. It should be warned that the sign will not be positive if $\dot{\boldsymbol{lnS}}$ is not assumed to be positive.

$$\frac{\partial\dot{\boldsymbol{\pi}}}{\partial\Phi} = \dot{\boldsymbol{lnS}} > 0$$

Evaluate at $\alpha = 2$.

$$\frac{\partial\dot{\boldsymbol{\pi}}}{\partial\Phi} = 2lnS\dot{\boldsymbol{lnS}} > 0$$

Therefore, the change of profit over time will rise with the increasing unit profit. Rearrange (1).

$$\dot{\boldsymbol{S}} = \Phi\alpha(lnS)^{\alpha-1}\dot{\boldsymbol{lnS}} + (lnS)^{\alpha}\dot{\boldsymbol{\Phi}} - (1+\delta)\dot{\boldsymbol{F}}\boldsymbol{X} - \dot{\boldsymbol{R}}\boldsymbol{D} - \dot{\boldsymbol{\pi}}$$

Find the direction of CSR over time along with variety of lnS on the horizontal axis. Assume $\dot{\boldsymbol{lnS}}$,$\dot{\boldsymbol{\Phi}}$, $\dot{\boldsymbol{F}}\boldsymbol{X}$, $\dot{\boldsymbol{R}}\boldsymbol{D}$, $\dot{\boldsymbol{\pi}}$ are constant and positive.

$$\frac{\partial\dot{\boldsymbol{S}}}{\partial lnS} = \Phi\alpha(\alpha-1)(lnS)^{\alpha-2}\dot{\boldsymbol{lnS}} + \alpha(lnS)^{\alpha-1}\dot{\boldsymbol{\Phi}}$$

Evaluate at $= 1$. It should be warned that the sign will not be positive if $\dot{\boldsymbol{lnS}}$ and $\dot{\boldsymbol{\Phi}}$ are not assumed to be positive.

$$\frac{\partial \dot{S}}{\partial lnS} = \dot{\Phi} > 0$$

Evaluate at $= 2$.

$$\frac{\partial \dot{S}}{\partial lnS} = 2\left[\Phi \dot{lnS} + lnS\dot{\Phi}\right] > 0$$

Therefore, the change of corporate social responsibility over time rises along with the value of logarithm of the social responsibility. These results construct a phase diagram as presented in Figure 4, 5 and 6.

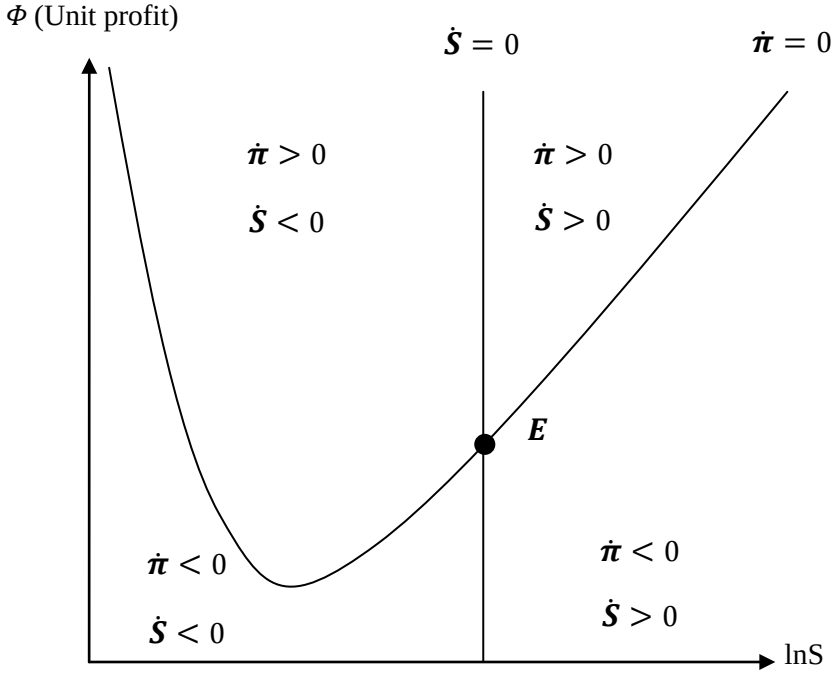

Figure 4: Phase diagram of the sustainability of profit and social responsibility

In the phase diagram (Figure 4 and 5), the steady state, the equilibrium in the long run, is indicated by point E. The steady state is at the intersection between the function of $\dot{\pi} = 0$ and $\dot{S} = 0$.

In the diagram, there are four areas. The upper-right area is mentioned as “Warm glow” area. The upper-left area is called “Frozen” area. The bottom-right area is named “Charitable” area and the bottom-left area can be viewed as “Decay” area.

In the warm glow area, the streamline force a firm to go toward only sustainable profit but far away from sustainable CSR. However, to achieve the sustainable profit, the firm has to spend the expenditure for CSR more and more. In the frozen area, the streamline leads a firm to achieve only sustainable profit too. In this case, the CSR will be shrunk. The firm will contribute less to the society. In the charitable area, a firm spends too much on CSR. The streamline will bring the firm to a high level of CSR but bankruptcy. In the decay area, the streamline will force the firm to end the business with both bankruptcy and zero responsibility to society.

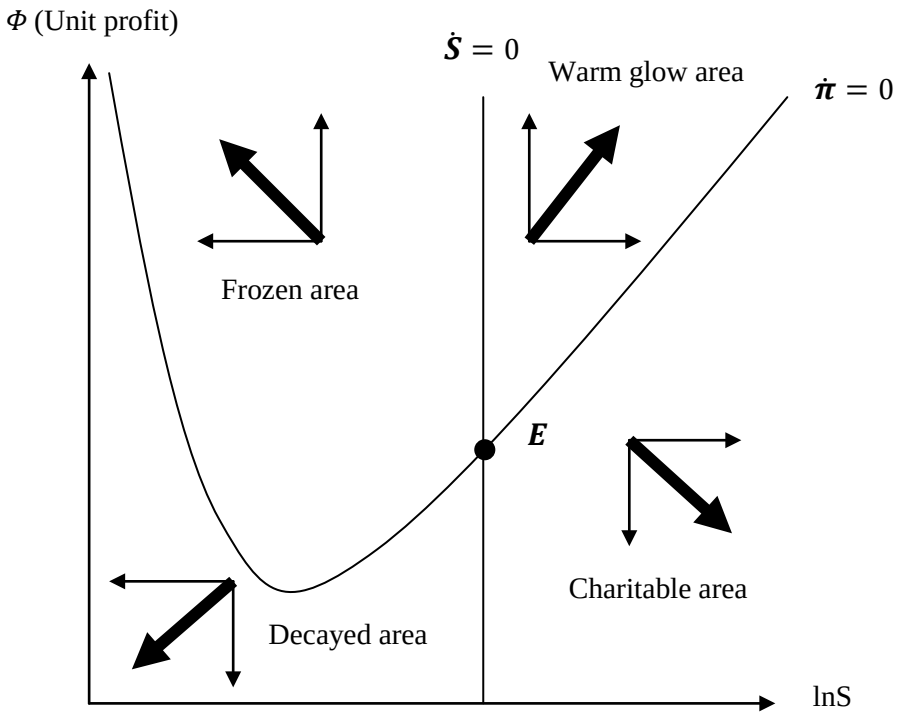

Figure 5: Phase diagram with stream lines and area classification.

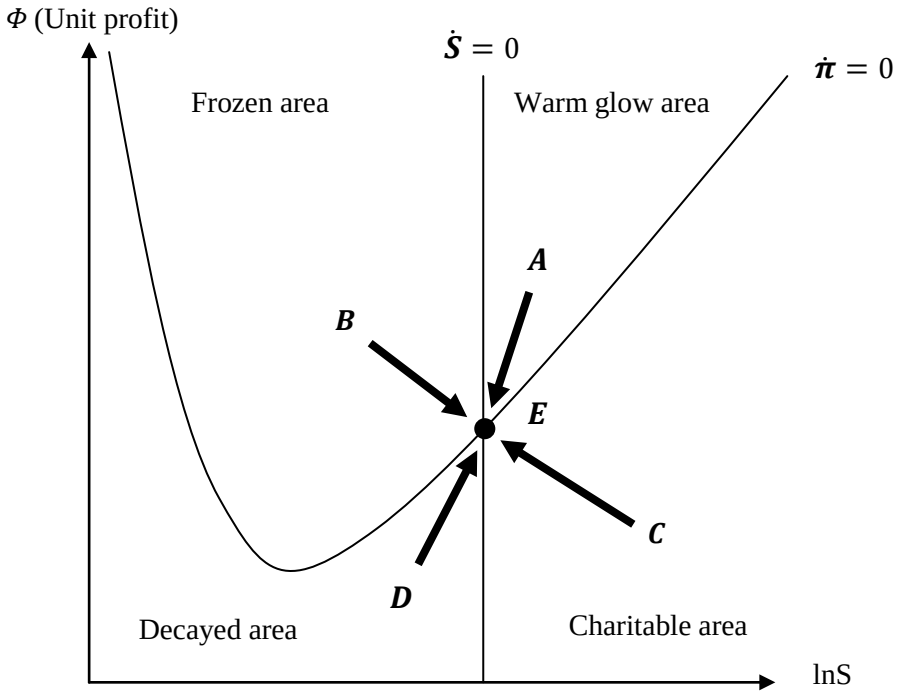

Figure 6: Policy manipulation to move the firm toward the sustainability of profit and social responsibility.

To achieve the twin sustainability of profit and social responsibility at the same time, a firm needs to manipulate its profit policy and CSR policy. A firm located in the warm glow area (point A) should reduce both the unit profit and CSR expenditure to lower the levels to point E. This seems that the firm has to reduce its role in society and also reduce its private benefit, but it will yield a better solution with both sustainability of profit and social responsibility in the long-run (Figure 6). Firms in other areas (point B, C and D) can also set the policies accordingly to achieve this twin sustainability.

4. Discussions

A firm cannot automatically achieve both sustainable profit and social responsibility at the same time. It needs policy manipulation and well-planned adjustment to move the business to the twin sustainability. Without manipulation, a firm located in warm glow area is expected to achieve only sustainable profit but it cannot easily stop spending for social responsibility. A firm located in frozen area can achieve only sustainable profit too but it has to shrink its social responsibility. The society will enjoy a firm located in charitable area when it spends too much for social responsibility and face the risk of bankruptcy. When the firm is located in the decayed area, it will face the downfalls of both the profit and expenditure for social responsibility.

An interesting point is that it can be seen that a firm in decayed area can avoid the bankruptcy by boosting up both the unit profit and CSR expenditure. This confirms D'Amato, Henderson and Florence (2009) that CSR can yield sustainable business too.

One result supports the principle of sufficiency economy. A firm with high unit profit and high CSR expenditure does not end up with sustainability in both the profit and social responsibility. It has to slow its pace to achieve the point by reducing both the unit profit and CSR expenditure. This confirms the idea that "Too much is not good" as presented mathematically in Suriya (2011). It is clear that not only a man but also a firm should live or operate in the middle path due to the Buddhist Economics thought and the philosophy of sufficiency economy.

5. Conclusions

This study constructs a model of sustainable profit and corporate social responsibility (CSR) using phase diagram. It begins the analysis with the profit function of a firm which includes traditional costs, research and development expenditure and expenditure for social responsibility. It defines the sustainability of profit over time as the zero growth of profit over time. It also defines the sustainability of corporate social responsibility by the constant CSR expenditure over time. It finds a function for the sustainable profit and another function of sustainable CSR. To achieve both targets at the same time, the steady state is located at the intersection of both graphs. It figures out the streamline of profit and CSR along with the change of unit profit and logarithm of CSR. With these four different streamlines, it classifies four areas in the diagram which are warm glow area, frozen area, charitable area and decayed area.

For the major result, it indicates that a firm cannot achieve both targets automatically without policy manipulation. The firm should adjust its profit policy and corporate social responsibility policy according to its location in the phase diagram. The study also supports the effect of corporate social responsibility on sustainable business. It also supports the ideology of middle path in Buddhist Economics and the philosophy of sufficiency economy

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