



## **Value at Risk Analysis of Gold Price Returns Using Extreme Value Theory**

Kittiya Chaithep, Songsak Sriboonchitta,  
Chukiat Chaiboonsri, and Pathairat Pastpipatkul

*Faculty of Economics, Chiang Mai University*

### **ABSTRACT**

There are many approaches to evaluate the return. However, Extreme Value Theory is the right method to analysis Value at Risk of Gold Price Return. The method is covered to the block maxima and the Peak over Thresholds modeling. This study uses a daily gold price in US dollar over the period of January 1, 1985 to August 31, 2011. The purpose is to evaluate a value at risk of the daily gold price return. It is very useful to manage a risk allocation in portfolio. Moreover, the paper is included to the return level forecasting in the next 20-years.

### **1. Introduction**

Gold is a major asset in terms of investment all over the world because it has served as one of the most stable monetary standards which have played a crucial role in the global economy. However, gold prices have been fluctuating since 1914. Historically, the United States has been determining the price of gold. One ounce of gold was fixed at an estimated price of \$20.67 US dollar for many decades until 1934. Subsequently the price of gold was raised to about \$35 US dollar per ounce. In 1968, a two-tiered pricing structure was established, and by 1975 the price of gold was allowed to fluctuate (L.S.Wynn, 2011). Presently, the price of gold has reached to about \$1,900 US dollar per ounce. Fluctuating gold prices has given rise to a growing speculation in gold.

The following chart shows annual average gold prices from 1914 to 2009. However, due to geo-political events between 1970 and 1990, a dramatic rise and fall in gold prices were caused by certain events such as Russian invasion of Afghanistan in Dec 1979, Iran hostage crisis and, a host of strong and unconventional policy actions and market events (the Fed under Volcker increased fed funds rate from 13% to 20% for a short period in Q1 1980, and Hunt's brothers silver market cornering failed due to their inability to meet a margin call during falling silver prices in March 1980, exacerbating the fall in precious metals). All of which events resulted in the roller coaster gold prices in 1980. (True North's Instablog, 2010).

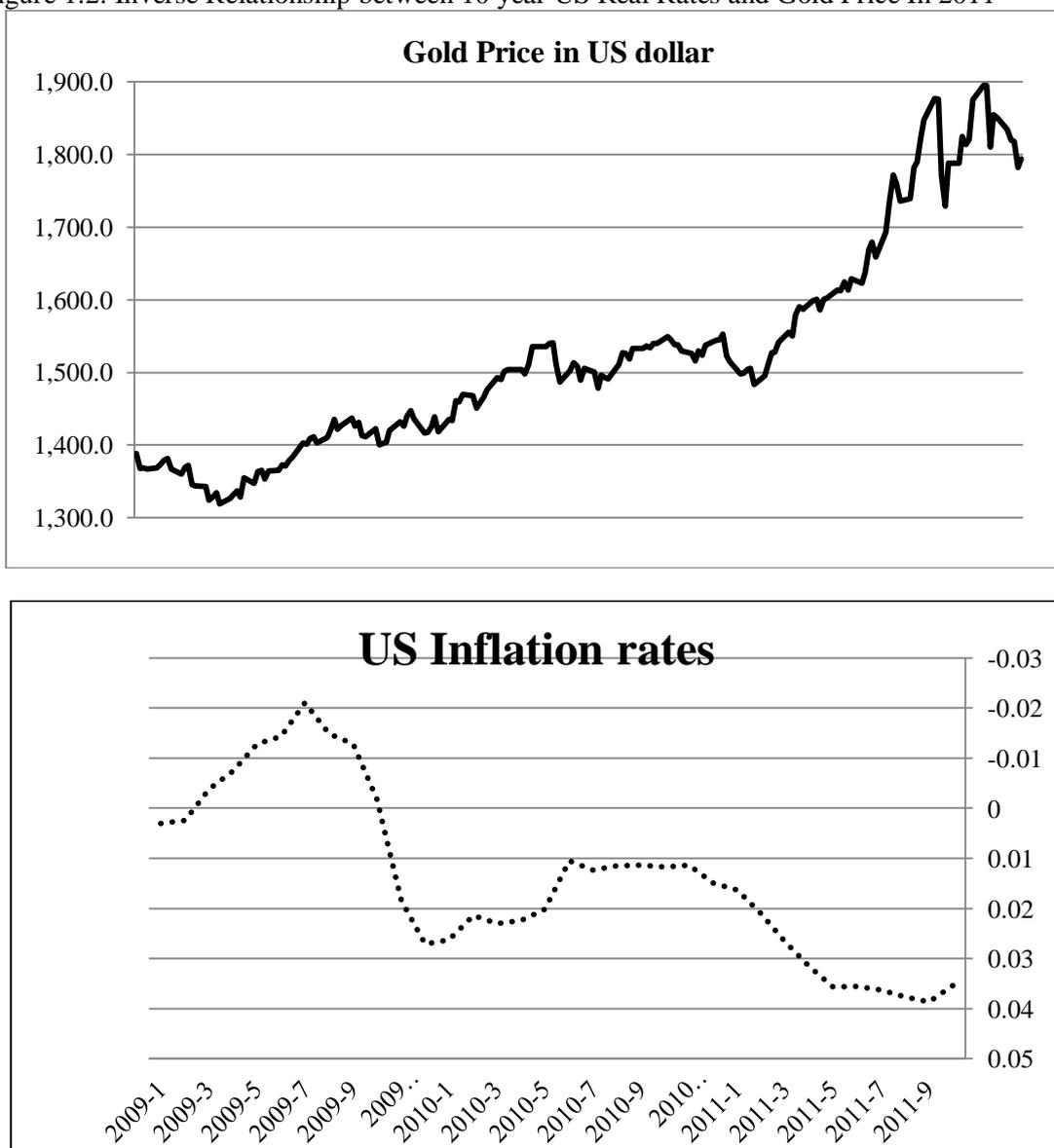
Figure 1.1: Average Annual Gold Prices in US dollar from 1900 to 2010



Source: Reuters DataStream, World Gold Council

Comparing gold prices to US real rate, the basic fundamentals in this inverse relationship are that when US monetary policy is looser, real rates fall and therefore demand for gold rises. Figure 2 shows that US real rates are in relation to gold prices, hence gold prices have an inverse relationship to US real rates. However if the US does embark on further monetary easing, or market expectations of easing increase, then US real interest rates could fall still further, implying an even higher gold price (Bob Kirtley, 2011).

Figure 1.2: Inverse Relationship between 10 year US Real Rates and Gold Price In 2011



Source:Falling Inflation, the Most Bullish Sign for Gold, 2011

Moreover, figure 1.3 shows how gold index and S&P500 index fluctuated in the last decade. The index of gold prices in US is more likely to increase than the index of S&P 500 from 2001 to 2011, using a base index as of January, 1 2001. It can be concluded that the rate of return on gold price is higher than the rate of return in S&P 500.

Figure 1.3: Gold and S&amp;P 500 in US (1 Jun 2001=100)



Source: World Gold Council

According to Table 1.1, the performance on various assets in US; Gold (US\$/oz), DJ UBS Comdty Index (Dow Jones-UBS Commodity Indexes), Brent crude oil (US\$/bbl), BarCap US Tsy Agg (The Barclays Capital Aggregate Bond Index), BarCap US High Yield (The Barclays Capital U.S. High Yield), S&P 500, the total return indices of gold is more interesting than other assets, it gives a -2.3% of 1-month investing, a 4.8% of 3-month investing, a 6.8% of 6-month investing, a 20.8% of 1-year investing, a 62.1% of 3-year investing, 150.2%, a 17.5% of 3-year CAGR (compounded annual growth rate) investing and a 20.1% of 5-year CAGR investing, respectively. Comparing to the total return indices of S&P 500, it gives only one more return at 30.7% of 1-year investing, others are less than gold return.

Table 1.1: Performance on various assets in US

	Gold (US\$/oz)	DJ UBS Comdty Index	Brent crude oil (US\$/bbl)	BarCap US Tsy Agg	BarCap US High Yield	S&P 500
1-month	-2.3%	-5.0%	-4.3%	-0.3%	-1.0%	-1.7%
3-month	4.8%	-6.7%	-4.8%	2.4%	1.1%	0.1%
6-month	6.8%	-0.9%	20.8%	2.6%	5.1%	6.0%
1-year	20.8%	25.9%	51.2%	2.2%	15.6%	30.7%
3-year	62.1%	-31.5%	-19.8%	16.1%	43.1%	10.3%
5-year	150.2%	1.3%	53.2%	35.6%	56.7%	15.4%
3y *CAGR	17.5%	-11.9%	-7.1%	5.1%	12.7%	3.3%
5y CAGR	20.1%	0.3%	8.9%	6.3%	9.4%	2.9%

\*CAGR = compounded annual growth rate (i.e., the geometric average rate of return over the corresponding period).

Source: Barclays Capital, World Gold Council; calculations based on total return indices unless not applicable.

As a result, table 1.2 indicates that the percentage change of demand for investment is at the highest, as estimated 36% in 12-month ending September 2011 to 12-month ending September 2010. This rise may be the result of the percentage increase of total bar and coin demand in 2008 to 2009, which is a proportion of 74%. However, gold demand for jewelry is at the highest at a \$79,399 US in 2010. Therefore, gold demand for jewelry, Technology and investment throughout in 2010 was on the rise, excepting Exchange Traded Funds and similar products.

Table 1.2: Gold Demand<sup>1</sup> (US million)

	2009	2010	Q3'11 vs Q3'10 % chg	4-quarter % chg <sup>2</sup>
<b>Jewelry</b>	56,695	79,399	24	31
<b>Technology</b>	12,811	18,363	39	30
Electronics	8,595	12,867	40	32
Other industrial	2,568	3,579	40	31
Dentistry	1,648	1,916	26	15
<b>Investment</b>	43,555	59,730	84	36
Total bar and coin demand	24,264	45,254	79	74
Physical bar demand	15,104	33,409	84	91
Official coin	7,319	8,367	87	33
Medals/imitation coin	1,841	3,477	14	42
ETFs and similar products <sup>3</sup>	19,291	14,476	119	-66
<b>Gold demand</b>	113,061	157,492	48	32

<sup>1</sup>: Gold demand excluding central banks

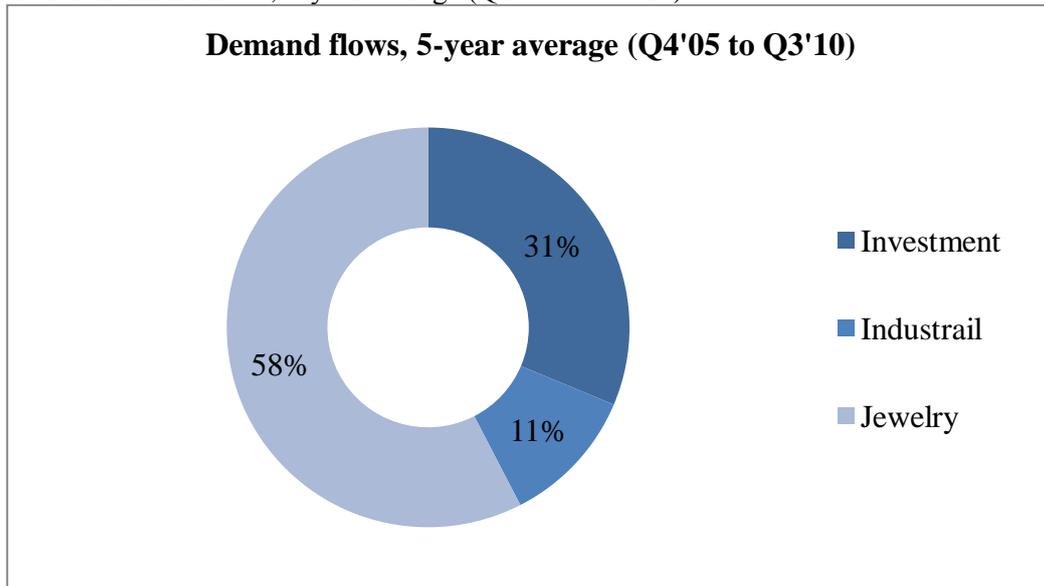
<sup>2</sup>: Percentage change, 12 months ended Sep 2011 vs 12 months ended Sep 2010.

<sup>3</sup>: Gold Exchange Traded Funds and similar products including: Gold Bullion Securities (London), Gold Bullion Securities (Australia), SPDR Gold Shares (formerly streetTRACKS Gold Shares), NewGold Gold Debentures, iShares Comex Gold Trust, ZKB Gold ETF, GOLDIST, ETF Securities Physical Gold, ETF Securities (Tokyo), ETF Securities (NYSE), XETRA-GOLD, Julius Baer Physical Gold, Central Gold ETF, Credit Suisse Xmtch and Dubai Gold Securities.

Source: LBMA, Thomson Reuters GFMS, World Gold Council

In addition, figure 1.4 shows demand flows in 5-year average of gold during 2005 to 2010. Jewelry takes accounts for over two-thirds of gold demand, which is around \$55 billion US, making it one of the world's largest categories of consumer goods. Second, a portion of investment demand is transacted in the over-the-counter market, therefore not easily measurable. However, there's no doubt that investment demand in gold has increased considerably in recent years, because the last five years to the end of 2009 saw an increase in value terms of around 119%. Last, Industrial, medical and dental technology accounts for around 12% of gold demand.

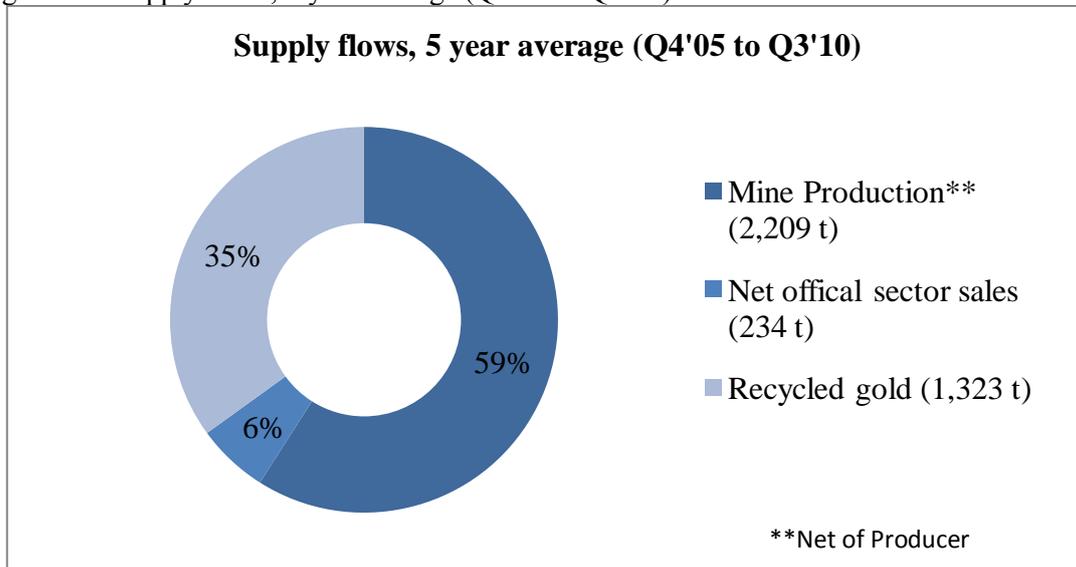
Figure 1.4: Demand flows, 5-year average (Q4'05 to Q3'10)



Source: World Gold Council

Moreover, figure 1.5 shows a proportion of gold flows. Mine production takes an account for 59% of total because there are several hundred gold mines operating worldwide ranging in scale from minor to enormous. Second, Recycle gold takes an account for 35% and 6% for net official sector sales.

Figure 1.5: Supply flows, 5 year average (Q4'05 to Q'310)



Source: World Gold Council

As was mentioned above, gold prices have significantly increasing all the time. However, gold prices have experienced both positive and negative side depending on the different events. To the most beneficial investment in gold, this study has investigated an evaluation of value at risk of gold price return at a given period.

One of the powerful instruments in financial market is Value at Risk estimator (VAR). Value at Risk has been established as a standard tool among financial institutions to depict the downside

risk of a market portfolio. It measures the maximum loss of the portfolio value that will occur over a given at some period at some specific confidence level due to risky market factors<sup>1</sup>. Moreover, VAR is a statistical measure the maximal possible losses which can be incurred in investment activities and losses that surpass the value of the Value at risk happen only with a certain probability<sup>2</sup>

The question for Value at risk is that what is the most we can lose on gold investment. Value at Risk tries to provide an answer, at least within a reasonable bound. This approach is a statistical measure the maximal possible losses which can be incurred in investment activities and losses that surpass the value of the Value at risk happen only with a certain probability (Linsmeier et al., 2000). It estimates the future distribution of returns. This could result in the holding of excessive amounts of cash to cover losses. Value at risk statistic has three components namely: a time period, confidence level and a loss amount (or loss percentage), for examples the question:

- What is the most I can expect to lose in dollars over the next month with a 95% or 99% level of confidence?
- What is the maximum percentage I can expect to lose over the next year with 95% or 99% confidence?

With many different approaches and models, namely, The Historical Simulation, The Variance-Covariance Method, Monte Carlo Simulation, Martin Odening and Jan Hinrichs<sup>3</sup>, investigate by using Extreme Value Theory to Estimate Value-at-Risk, stated that this article examines problems that may occur when conventional Value-at-Risk (VAR) estimators are used to quantify market risks in an agricultural context. For example, standard Value at risk methods, such as variance-covariance method or historical simulation, can fail when the return distribution is fat tailed. This problem is aggravated when long-term Value at risk forecasts is desired. Extreme Value Theory is proposed to overcome these problems. Neftci (2000) found that the extreme distribution theory fit well for the extreme events in financial markets. Bali (2003) determines the type of asymptotic distribution for the extreme changes in U.S. Treasury yields. In his paper, the thin-tailed Gumbel and exponential distribution are worse than the fat-tailed Fréchet and Pareto distributions. Gençay and Selçuk (2004) investigate the extreme value theory to generate VAR estimates and study the tail forecasts of daily returns for stress testing.

Based on those applications of extreme value theory, it is the appropriate model that matches the purpose of this study best. This paper focuses on risk evaluation of gold price return and the tail distribution of extreme events in gold price returns (in US dollars).

## 2. Extreme Value Theory and Statistical Approaches

---

<sup>1</sup> Martin Odening and Jan Hinrichs, 2003 Quoted in Jorion, P., 1997.

<sup>2</sup> Vladimir Djakovic\*, Goran Andjelic, and Jelena Borock, 2010:340 Quoted in Linsmeier TJ, Pearson ND, 2000.

<sup>3</sup> Martin Odening, Jan Hinrichs, 2003. "Using extreme value theory to estimate value-at-risk." *Agricultural Finance Review* 63: 55 – 73

This paper is used Extreme Value Theory and Statistical Approaches. Extreme value theory relates to the asymptotic behavior of extreme observations of a random variable. It provides the fundamentals for the statistical modeling of rare events, and is used to compute tail risk measures. Researchers have contributed abundant theoretical discussion on EVT such as Embrechts et al. (1997), Reiss and Thomas (1997), and Coles (2001). Modeling Extreme Value Theory, there are two ways if identified extremes in data.

This paper is considered a random variable which may represent daily losses or returns. The first approach considers the maximum (or minimum) the variable takes in periods. The second approach focuses on the largest value variable over some high threshold.

### 2.1 Block Maxima or Generalized Extreme Value Distribution (GEV)

This approach is the one of studying the limiting distributions of the sample extreme, which is presented under a single parameterization. In this case, extreme movements in the left tail of the distribution can be characterized by the negative numbers (Jiahn-Bang Jang, 2007)

Let  $X_i$  be the negative of the  $i$  th daily return of the gold prices between day  $i$  and day  $i-1$ . Define

$$X_i = -(\ln P_i - \ln P_{i-1})$$

where  $P_i$  and  $P_{i-1}$  are the gold prices of day  $i$  and day  $i-1$ . Suppose that  $X_1, X_2, \dots, X_n$  be iid random variables with an unknown cumulative distribution function (CDF)  $F(x) (= \Pr(X_i \leq x))$ . Extreme values are defined as maxima (or minimum) of the  $n$  independently and identically distributed random variables  $X_1, X_2, \dots, X_n$ .

Then, let  $X_n$  be the maximum negative side movements in the daily gold prices returns, that is

$$X_n = \max(X_1, X_2, \dots, X_n)$$

Since the extreme movements are the focus of this study, the exact distribution of  $X_n$  can be written as

$$\Pr(X_n \leq a) = \Pr(X_1 \leq a, X_2 \leq a, \dots, X_n \leq a) = \prod_{i=1}^n F(a) = F^n(a)$$

In practice the parent distribution  $F$  is usually unknown or not precisely known. The empirical estimation of the distribution  $F^n(a)$  is poor in this case. Fisher and Tippet (1928) derived the asymptotic distribution of  $F^n(a)$ . Suppose  $\mu_n$  and  $\sigma_n$  are sequences of real number location and scale measures of the maximum statistic  $X_n$ . Then the standardized maximum statistic

$$Z_n^* = \left( \frac{X_n - \mu_n}{\sigma_n} \right) \quad (1)$$

Converges to  $z = (x - \mu) / \sigma$  which has one of three forms of non-degenerate distribution families such as

$$\begin{aligned} H(z) &= \exp\{-\exp[-z]\}, -\infty < z < \infty \\ H(z) &= \exp\{-Z^{-1/\xi}\}, z > 0 = 0, \text{ else} \\ H(z) &= \exp\{-[Z]^{-1/\xi}\}, z > 0 = 1, \text{ else} \end{aligned} \quad (2)$$

These forms go under the names of Gumbel, Frechet, and Weibull respectively. Here  $\mu$  and  $\sigma$  are the mean return and volatility of the extreme values  $x$  and  $\xi$  is the shape parameter or called  $1/\xi$  the tail index of the extreme statistic distribution. With  $\xi = 0$ ,  $\xi > 0$ ,  $\xi < 0$  represent

Gumbel, Frechet, and Weibull types of tail behavior respectively. In fact Gumbel, Frechet, and Weibull types can be fit for exponential, long, and short tails respectively.

Embrechts and Mikosch (1997) proposed a generalized extreme value (GEV) distribution which included those three types and can be used for the case stationary GARCH processes. GEV distribution has the following form

$$\begin{aligned}
 H_{\xi}(X; \mu, \sigma) &= \exp \left\{ -\frac{\exp[-x - \mu]}{\sigma} \right\}, -\infty \leq x \leq \infty; \xi = 0 \\
 &= \exp \left\{ -\left[ 1 + \xi(x - \mu)/\sigma \right]^{-\frac{1}{\xi}}, 1 + \frac{\xi(x - \mu)}{\sigma} > 0; \xi \neq 0 \right. \quad (3)
 \end{aligned}$$

Then, suppose that block maxima  $B_1, B_2, \dots, B_k$  are independent variables from a GEV distribution, the log-likelihood function for the GEV, under the case of  $\xi \neq 0$ , can be given as

$$\ln L = -k \ln \sigma - \left( 1 + \frac{1}{\xi} \right) \sum_{i=1}^k \ln \left\{ 1 + \xi \frac{(B_i - \mu)}{\sigma} \right\} - \sum_{i=1}^k \left\{ 1 + \xi \frac{(B_i - \mu)}{\sigma} \right\}^{-\frac{1}{\xi}} \quad (4)$$

For the Gumbel type GEV form, the log-likelihood function can be written as

$$\ln L = -k \ln \sigma - \sum_{i=1}^k \frac{(B_i - \mu)}{\sigma} - \sum_{i=1}^k \exp \left\{ \frac{(B_i - \mu)}{\sigma} \right\} \quad (5)$$

As Smith (1985) declared that, for  $\xi > 0.5$ , the maximum likelihood estimators, for  $\xi$ ,  $\mu$ , and  $\sigma$ , satisfy the regular conditions and therefore having asymptotic and consistent properties. The number of blocks,  $k$  and the block size form a crucial tradeoff between variance and bias of parameters estimation.

So, as it follows from (3), VaR at  $p$  level can be estimated as, (Marinelli and et al,2006)

$$\text{VaR}_{\text{GEV}} = \mu - \left( \frac{\sigma}{\xi} \right) \left[ 1 - (-\log p)^{-\xi} \right]$$

where  $\mu$ : location parameter

$\sigma$ : scale parameter

$\xi$ : shape parameter

$p$ : exceeded by the annual maximum in any particular year with probability.

### 2.2 Peak over threshold or Generalized Pareto Distribution (GPD)

Jiahn-Bang Jang, 2007 stated that Peaks over Thresholds (POT) method utilizes data over a specified threshold. Define the excess distribution as

$$F_h(x) = \Pr(X - h < x \mid X > h) = \frac{[F(x + h) - F(h)]}{1 - F(h)} \quad (6)$$

where  $h$  is the threshold and  $F$  is an unknown distribution such that the CDF of the maxima will converge to a GEV type distribution. For large value of threshold  $h$ , there exists a function  $\tau(h) > 0$  such that the excess distribution of equation (6) will approximated by the generalized Pareto distribution (GPD) with the following form

$$\begin{aligned}
 H_{\xi, \tau(h)}(x) &= 1 - \exp \left( -\frac{x}{\tau(h)} \right), \xi = 0 \\
 &= 1 - \left( 1 + \frac{\xi x}{\tau(h)} \right)^{-1/\xi}, \xi \neq 0 \quad (7)
 \end{aligned}$$

where  $x > 0$  for the case of  $\xi \geq 0$ , and  $\xi \geq 0 \geq x$ , and  $0 \leq x \leq \tau(h) / \xi$  for the case of  $\xi < 0$ . Define  $X_1, X_2, \dots, X_k$  as the extreme values which are positive values after subtracting threshold  $h$ .

For large value of  $h$ ,  $X_1, X_2, \dots, X_k$  is a random sample from a GPD, so the unknown parameters  $\xi$  and  $\tau(h)$  can be estimated with maximum likelihood estimation on GPD log-likelihood function.

Based on equation (6) and GPD distribution, the unknown distribution  $F$  can be derived as

$$F(y) = (1 - F(h))H_{\xi, \tau(h)}(x) + F(h) \quad (8)$$

where  $y = h + x$ .  $F(h)$  can be estimated with non-parametric empirical estimator

$$\hat{F}(h) = \frac{n - k}{n}$$

where  $k$  is the number of extreme values exceed the threshold  $h$ . Therefore the estimator of (8) is

$$\hat{F}(h) = (1 - \hat{F}(h)) \hat{H}(x; \hat{\xi}, \hat{\tau}(h)) + \hat{F}(h) \quad (9)$$

where  $\hat{\xi}$  and  $\hat{\tau}(h)$  are mle of GPD log-likelihood. High quantile VaR and expected shortfall can be computed using (9). First, define  $F(\text{VaR}_q) = q$  as the probability of distribution function up to  $q^{\text{th}}$  quantile  $\text{VaR}_q$ . Therefore

$$\widehat{\text{VaR}}_q = \hat{F}^{-1}(q) = h + \frac{\hat{\tau}(h) \left\{ \left[ \left( \frac{n}{k} \right) (1-q) \right]^{-\hat{\xi}} - 1 \right\}}{\hat{\xi}} \quad (10)$$

Or it can be written as Generalized Pareto distribution estimating the value at Risk (Ayse Kisacik, 2006),

$$\text{VaR}_{\text{GPD}} = u + \left( \frac{\sigma}{\xi} \right) \left[ \left[ \frac{n}{N_u} \alpha \right]^{-\xi} - 1 \right], \xi \neq 0$$

where  $u$ : threshold

$\sigma$ : scale parameter

$\xi$ : shape parameter

$\alpha$ :  $1-p$

$N_u$ : number of exceedances

$n$ : sample size

Next, given that  $\text{VaR}_q$  is exceeded, define the expected loss size, so called Expected shortfall (ES); the average of all losses which are greater or equal than  $\text{VaR}$ , as

$$\text{ES}_q = E(X | X > \text{VaR}_q) = \text{VaR}_q + E(X - \text{VaR}_q | X > \text{VaR}_q) \quad (11)$$

From (10), (Jiahn-Bang Jang, 2007)  $\widehat{\text{ES}}_q$  can be computed using  $\widehat{\text{VaR}}_q$  and the estimated mean excess function of GPD distribution. Therefore,

$$\widehat{\text{ES}}_q = \frac{\widehat{\text{VaR}}_q}{1 - \hat{\xi}} + (\hat{\tau}(h) - \hat{\xi}h)/(1 - \hat{\xi})$$

### 3. Literature review

### 3.1 Value at Risk

In term of evaluation in Value at Risk, Jaroslav Baran and Jiří Witzany (2010) applied EVT in estimating low quantiles of P/L distribution and the results are compared to common VAR methodologies. The result confirms that EVTGARCH is superior to other methods. Gençay and Selçuk (2004), they investigate the extreme value theory to generate Value at Risk to estimate and study the tail forecasts of daily returns for stress testing. Then, Bali (2003) studies how to estimate volatility and Value at Risk by an Extreme Value Approach and determines the type of asymptotic distribution for the extreme changes in U.S. Treasury yields. In this paper, the thin-tailed Gumbel and exponential distribution are worse than the fat-tailed Fréchet and Pareto distributions.

In the analysis of Stelios Bekiros and Dimitris Georgoutsos (2003) conduct a comparative evaluation of the predictive performance of various Value-at-Risk (VAR) models. Both estimation techniques are based on limit results for the excess distribution over high thresholds and block maxima respectively. The results we report reinforce previous ones according to which some “traditional” methods might yield similar results at conventional confidence levels but at very high ones the EVT methodology produces the most accurate forecasts of extreme losses. Moreover, Yasuhiro Yamai and Toshinao Yoshida (2002) investigate the comparison of value-at-risk (VAR) and expected shortfall under market stress. The paper found that First, VAR and expected shortfall may underestimate the risk of securities with fat-tailed properties and a high potential for large losses. Second, VAR and expected shortfall may both disregard the tail dependence of asset returns. Third, expected shortfall has less of a problem in disregarding the fat tails and the tail dependence than VAR does.

### 3.2 Extreme Value Theory Approach

Extreme Value Theory is most use in evaluation of Value at Risk in Financial market. Martin Odening and Jan Hinrichs<sup>4</sup> (2010), who investigate on Using Extreme Value Theory to Estimate Value-at-Risk, stated that this article examines problems that may occur when conventional Value-at-Risk (VAR) estimators are used to quantify market risks in an agricultural context. For example, standard Value at risk methods, such as variance-covariance method or historical simulation, can fail when the return distribution is fat tailed. This problem is aggravated when long-term Value at risk forecasts is desired. Extreme Value Theory is proposed to overcome these problems. For a stock market study, Vladimir Djakovic, Goran Andjelic, and Jelena Borocki (2010) investigate the performance of extreme value theory (EVT) with the daily stock index returns of four different emerging markets. Research results according to estimated Generalized Pareto Distribution (GPD) parameters indicate the necessity of applying market risk estimation methods and it is clear that emerging markets such as those of selected emerging markets have unique characteristics.

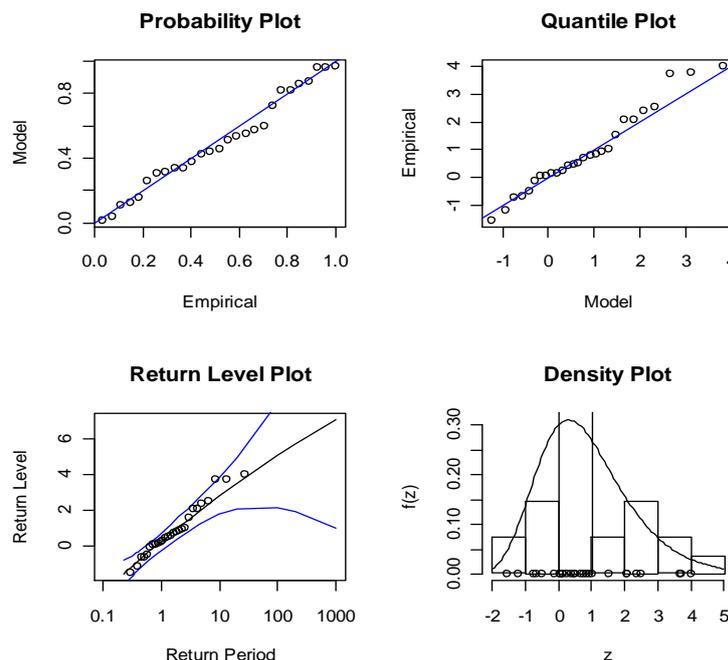
---

<sup>4</sup> Professor of Farm Management and PhD candidate, respectively. Department of Agricultural Economics, Humboldt University Berlin, Germany.

In the analysis of gold price return, The study of Jiahn-Bang Jang) 2007( has been examined to illustrate the main idea of extreme value theory and discuss the tail behavior. The results show that GPD model with threshold is a better choice. Also, Blake LeBaron and Ritirupa Samanta) 2004 (investigate that how to apply Extreme Value Theory (EVT) to construct statistical tests. The result shows that EVT elegantly frames the problem of extreme events in the context of the limiting distributions of sample maxima and minima. In financial market study, Neftci (2000) found that the extreme distribution theory fit well for the extreme events in financial markets. Moreover, Alexander J. McNeil (1999) investigates about Extreme Value Theory for Risk Managers. In this paper, the tail of a loss distribution is of interest, whether for market, credit, operational or insurance risks, the POT method provides a simple tool for estimating measures of tail risk. Also, Rootzen and Tajvidi (2006) define the multivariate Generalized Pareto distributions and to prove that this definition indeed is the right one. Since the Peak over Threshold method uses more of the data it can sometimes result in better estimation precision than the block maxima method.

#### 4. The empirical result of Generalized Extreme values (GEV): Gold Price Return

Figure 4.1: Diagnostic plots for GEV fit to the Gold Price Return.



The variance diagnostic plots for assessing the accuracy of the GEV model fitted to the Gold Price Return are shown in Fig. 4.1. The probability plot is fitted the model as near-linear but the quintiles plot are fitted the model as near-linear in the beginning but it outwards in finally. It means that the distribution of model is not good in the long term. The return level curve asymptotes to a finite level as a consequence of the positive estimate of  $\xi$ , though since the estimate is close to zero. The estimated curve is closed to linear. The density estimate seems consistent with the histogram of the data. Maximization of the GEV log-likelihood for these data leads to the estimate:  $(\mu, \sigma, \xi) =$

(0.1948, 1.1907, -0.0066). The shape parameter is short tailed because  $\xi$  is less than zero. Taking square roots, the standard errors are 0.25969, 0.18735 and 0.15581 for  $\mu$ ,  $\sigma$  and  $\xi$  respectively.

Table 4.1: The Summary results of gold price return in next 20 year (2012-2032) based on Generalize Extreme Value Analysis

Table 4.1 presents the gold price return in US dollar based on Generalize Extreme Value Analysis in during period of 1985-2011 to forecast the gold price return of the next twenty year (2012-2032). For the United State of America the extreme value of gold price return will be increased since 2012-2032. In the 2017 the extreme values of gold price return will be 1.9514% (1.0931%, 3.4273%) at the significant level of 99%.

Item	$\mu$	$\sigma$	$\xi$	5-year	10-year	20-year
Gold Price Return	0.1948	1.1907	-0.0066	1.9514	2.7574	3.5008
95%				(1.2820,2.9558)	(1.9588,4.2629)	(2.5280,5.6256)
99%				(1.0931,3.4273)	(1.7505,4.7360)	(2.3034,6.2933)

In 2022 the extreme values of gold price return will be 2.7574% (1.7505%, 4.7360%) at the significant level of 99%. In 2032 the extreme values of gold price return will be 3.5008% (2.3034%, 6.2933%) at the significant level of 99%.

When we compute the value of parameters in VAR equation, we got 6.5461%. It means that the extreme value in tomorrow's loss will be is 6.5461% in a day at the significant level of 99%.

## 5. The empirical result of Generalize Pareto Distribution (GPD): Gold Price Return

Figure 5.1: Mean residual life plot for daily gold price return

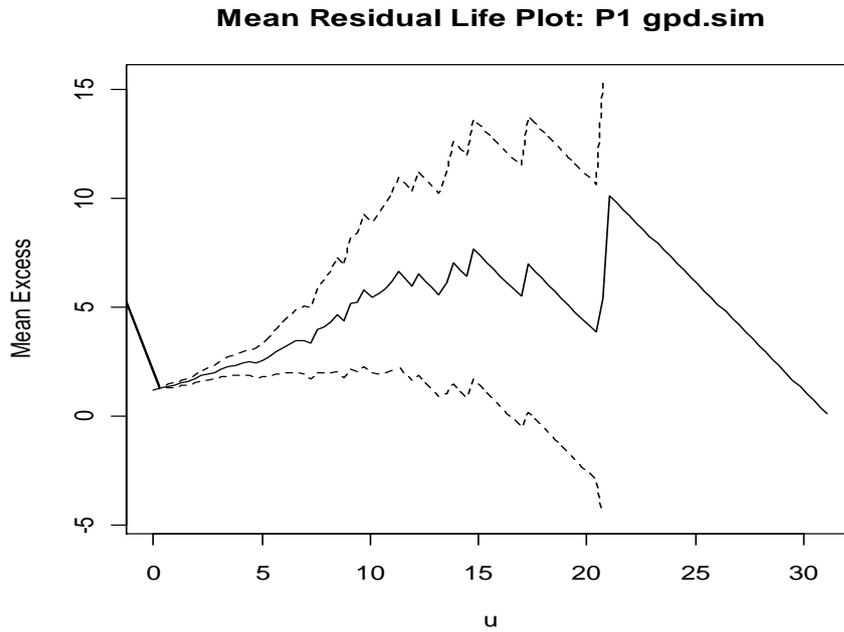


Figure 5.2: Parameter estimates against threshold for daily gold price return

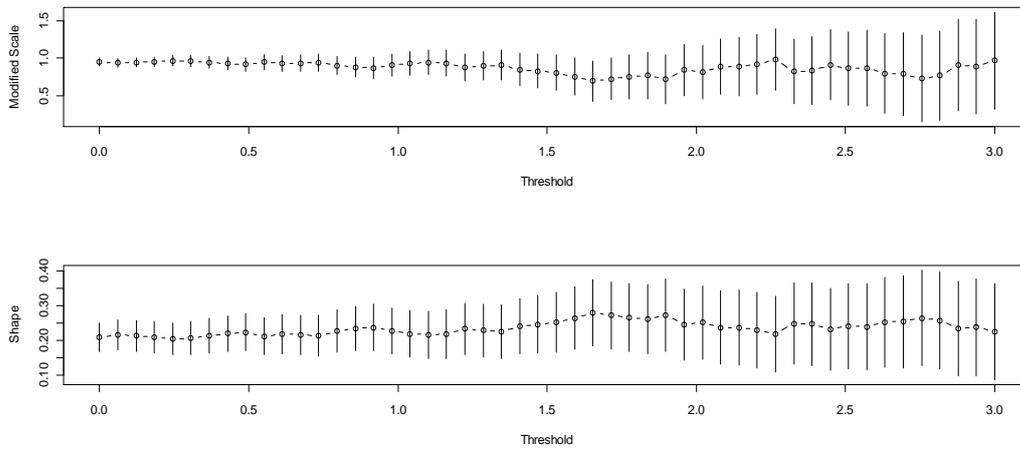


Figure 5.3: Diagnostic plots for threshold model fitted to the Gold Price Return

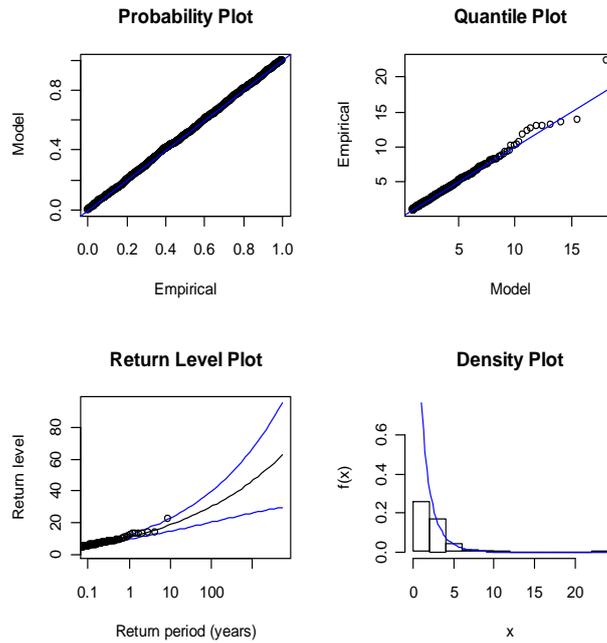


Figure 5.1 shows the mean residual life plot with approximate 95% confidence intervals for the daily gold price return. The graph appears to curve from  $u=0$  to  $u \approx 5$ , beyond which it is approximately linearity. Accordingly, it is better to conclude that there is some evidence for linearity around  $u=0$  to  $u=3$ . Then, figure 5.2 can show more obvious mean residual life plot for parameter estimate, which the selected threshold of  $u=1$  is reasonable. There are 1222 exceedances of the threshold  $u=1$  in the complete set of 3181 observations. Maximum likelihood estimates in this case are  $(\sigma, \xi) = (1.3173804, 0.1562862)$  with a corresponding maximized log-likelihood of 1750.1696. The shape parameter is fat tailed because  $\xi$  is greater than zero.

Diagnostic plots for threshold model are shown in Fig.5.3. The goodness-of-fit in the quantile plot is unconvincing because there has close to near-linear. Also, the return level also illustrates the large certainties that fit well to the model.

Table 5.1: The Summary results of gold price return in next 20 year (2012-2032) based on Generalize Pareto Distributions Analysis

Item	$\sigma$	$\xi$	5-year	10-year	20-year
Gold Price Return	1.314 7	0.157 9	16.045	18.7308	21.7239
95%			(13.533,19.8699)	(15.3960,23.9787)	(17.3851,28.7828)
99%			(12.927,21.4692)	(14.6082,26.2251)	(16.3810,31.8770)

Table 5.1 presents the gold price return in US dollar based on Generalize Pareto Distributions Analysis in during period of 1985-2011 to forecast the gold price return of the next twenty year (2012-2032). For the United State of America the extreme value of gold price return will be increased since 2012-2032. In the 2017 the extreme values of gold price return will be 16.045% (12.92709%, 21.46928%) at the significant level of 99%. In 2022 the extreme values of gold price return will be 18.7308% (14.60828%, 26.22517%) at the significant level of 99%. In 2032 the extreme values of gold price return will be 21.7239% (16.38103%, 31.87702%) at the significant level of 99%.

The log-likelihood estimates we obtain are  $\sigma = 1.3147$  and  $\xi = 0.1579$ . When we compute the value of parameters in VAR equation,  $\text{VAR}_{0.01}$  is 7.4867%. It means that the extreme value in tomorrow's loss will be is 7.4867% in a day at the significant level of 99%. If we invest \$1 million in gold return, we are 99% confident that our daily loss at worst will not exceed \$74,867 during one trade day. In other word, we are 1% confident that our daily loss will exceed 7.4867% or \$74,867 during one trade day if we have an investment of \$1 million in that market. Then, Expected Shortfall which is the average amount that is lost over a given day period, assuming that the loss is greater than the 99th percentile of the loss distribution (Artzner, et al., 1999), shows the result as 10.2748 % with 99% confidence intervals. It means that the average of those losses below a level of 99% confidence intervals in one-day period will be 10.2748%. If we invest \$1 million in gold return, we are 99% confident that our daily.

## 6. Conclusions

In financial market, there are many approaches to evaluate the Value at Risk of the asset return, such as Variance-covariance, Historical simulation, Extreme Value Theory. The asset price return has fluctuated all the time depending on the extreme events namely Stock price, Derivatives price, Commodity price, etc. Most asset prices are not usually a normal return distribution. They are quite a tailed-distribution of return. Since, Variance-covariance and Historical simulation are failed to fit a tail distribution. Therefore, Extreme Value Theory is the right method to analysis Value at Risk of asset because it can overcome this problem.

The most interesting commodity asset in this decade is gold, because gold price has been increasing all the time. Gold is a major asset in terms of investment all over the world because it has served as one of the most stable monetary standards which have played a crucial role in the global economy.

This study investigates the estimation a value at risk of gold price return and the forecast of next 20-years gold price return. Empirical research reveals that value of gold price return when modeled after Generalized Extreme Value (GEV) is that a maximum tomorrow's loss is 6.5461% at the significant of 99 percent confidence interval. It can be concluded that if we invest \$1 US million in gold price, we are 99% confident that the daily loss in 1-day period will not exceed to \$65,461 US. In other words, we are 1% confident that our daily loss will exceed \$65,461 US during one trade days.

Second, it is shown that the Generalized Pareto Distributions (GPD) corresponds to the tails of the return distributions well. Estimation of left tails at 0.999 percentile with 99

percent confidence interval show that it is possible to observe over 7.487 % loss in one day and an expected shortfall is 10.2748% in a single day. It means that if we invest \$1 million US dollar in gold price, we are 99% confident that the daily loss in 1-day period will not exceed to \$74,870 US. In other words, we are 1% confident that our daily loss will exceed \$74,870 US during one trade days. Expected shortfall is 10.2748% in a single day. As well as if we invest \$1 million in gold, we are 99% confident that the daily average amount loss over the 99th percentile of the loss distribution is \$102,748 US dollar.

Although the results from 2 different methods are quite similar, GPD gives a more accurate result for Value at Risk. Since GPD is more modern approach to extreme events and it uses more data for estimation. It also separates the data estimation of the left and right tail distribution to fit in with the model.

Further research will be divided into two parts: prior to the financial crisis in 2008 (Leman Brother) and post 2008 following crisis estimate Value at Risk of Gold price return. Also, the study will focus in both negative and positive return to compare the results.

## REFERENCES

- Alexander J. McNeil, 1999. **Extreme Value Theory for Risk Managers**. Departement Mathematik ETH Zentrum, CH-8092 Zurich.
- Artzner P, F Delbaen, J-M Eber and D Heath, 1999. **Coherent measures of risk** *Mathematical Finance* 9 (3), pp. 203-228. Risk management and financial institutions Prentice Hall.
- Ayse Kisacik, 2006. **High volatility, heavy tails and extreme values in value at risk estimation**. Institute of Applied Mathematics Financial Mathematics/ Life Insurance Option Program Middle East Technical University, Term Project.
- A. J. McNeil and R. Frey. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *J. Empirical Fin.*, 7:271–300, 2000.
- Bali, T. G., 2003. “An extreme value approach to estimating volatility and value at risk”, *Journal of Business*, 76:1 83-108.
- Baran, J., Witzany, J., 2010. **A Comparison of EVT and Standard VaR Estimations**, Faculty of Finance and Accounting, University of Economics.
- Coles, S., 2001. **An Introduction to Statistical Modeling of Extreme Values**. Springer-Verlag, London.
- C. Marinelli, S. d’Addona, S. T. Rachev and Department of Statistics and Applied Probability University of California, 2006. A comparison of some univariate models for Value-at-Risk and expected shortfall. **Working paper**, 2006
- Djakovic, V., Andjelic, G. and Borocki, J., 2010. **Performance of extreme value theory in emerging markets: An empirical treatment**. Faculty of Technical Sciences. University of Novi Sad.
- Embrechts, P., Klüppelberg, C, and Mikosch, T., 1997. **Modeling extremal events for insurance and finance**, Berlin and Heidelberg: Springer.
- Fisher, R. and Tippett, L., 1928. **Limiting forms of the frequency distribution of the largest or smallest member of a sample**, *Proceedings of the Cambridge Philosophical Society*, 24, 180-190.
- Gencay, R. and Selcuk, F., 2004. “Extreme value theory and value-at-risk: relative performance in emerging markets”, *International Journal of Forecasting*, 20:2, 287-303.

- H. Rootzen and N. Tajvidi, 2006. The multivariate generalized Pareto distribution. **Bernoulli**12, 917-930.
- Jiahn-Bang Jang, 2007. **An Extreme Value Theory Approach for Analyzing the Extreme Risk of the Gold Prices**, 97-109
- Kirtley, B., 2011. **Falling Inflation, The most Bullish Sign for Gold**. Available: <http://www.marketoracle.co.uk/Article31409.html>(November 18, 2011) [online]
- LeBaron, B. and Samanta, R., 2004. **Extreme value theory and fat tails in equity markets**.
- Linsmeier TJ, Pearson ND, 2000. "Value at Risk Mar/Apr 2000." **Finance, Journal**, 56(2): 47-67
- Liu, F., 2009. **Extreme Value Theory to Estimating Value at Risk**. Department of Statistics and Operations Research, University of North Carolina.
- L.S.Wynn, 2011. **What is the Historical Price of Gold**. Available: <http://www.wisegeek.com/what-is-the-historical-price-of-gold.htm> (October 14, 2011) [online]
- M. Gilli and E. Kellezi. *An application of extreme value theory for measuring financial risk*. Working paper, 2005.
- McNeil, A. (1997), **Estimating the tails of loss severity distributions using extreme value theory**, ASTIN Bulletin 27, 117-137.
- McNeil, A. (1999), **Extreme value theory for risk managers**, **Internal Modeling and CAD II, Risk Books**, page 93-113.
- Neftci, S. N., 2000. **Value at risk calculations, extreme events, and tail Estimation**. *Journal of Derivatives*, 7:3, 23-37.
- Odening, M., Hinrichs, J., 2010. **Using Extreme Value Theory to Estimate Value-at-Risk**. Department of Agricultural Economics, Humboldt University Berlin, Germany, 1-27
- Reiss, R.D., Thomas, M., 1997. **Statistical Analysis of Extreme Values with Applications to Insurance, Finance, Hydrology and Other Fields**. Birkhäuser Verlag, Basel.
- Smith, R. L., 1985. **Maximum likelihood estimation in a class of nonregular cases**, **Biometrika**, 72, 67-90.
- True North's Instablog, 2010. **An important history lesson: Gold's dramatic rise and fall in 1980s**. Available: <http://seekingalpha.com/instablog/492761-true-north/59248-an-important-history-lesson-gold-s-dramatic-rise-and-fall-in-1980s>(October 20, 2011)