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Comparison of sales forecasting models for an innovative agro-industrial product: Bass model versus logistic function

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ABSTRACT

This paper compares the accuracy of sales forecasting between Bass model (Bass, 1969) and Logistic function (Stoneman, 2010). It uses several ways to estimate the models; least squares with quadratic interpolation, least squares with quasi-Newton, maximum likelihood with quadratic interpolation and maximum likelihood with quasi-Newton. It applies the technique to an innovative agro-industrial product, feta cheese from buffalo milk. Then it compares the performance of the models by Mean Absolute Percentage Error (MAPE) of the out-of-sample test. It matches Bass model and Logistic function which are estimated by the same method and compare their performances. Moreover, it compares the best Bass model with the best Logistic function regardless of the estimation method. The results reveal that, in most pairs, Logistic function is superior than Bass model when the model uses the data between 7 to 24 months which MAPE of Logistic function are improved tremendously. However, the performance of the best Logistic function is insignificantly different to that of Bass model.

Keywords: Innovative product, agro-industrial product, sales forecasts, Bass model, Logistic function.

JEL classification: C53, O31, M31

1. Introduction

Sales forecasting of innovative agro-industrial product is important for the establishment of the further development of the product. The more accurate forecast will guide the producer to the more efficient operation. In forecasting, there are several functional forms or models to use. Bass model introduced by Bass (1969) is the most famous one. However, modern literature such as Stoneman (2010) suggested that the Logistic function may be suitable for the forecasts. Therefore, this paper will find out which functional form is better in sales forecasts of feta cheese from buffalo milk.

2. Conceptual framework and literature review

2.1 Bass model

Bass (1969) and Srinivasan and Mason (1986) indtroduced a functional form to forecast sales of new products as follows:

$$V = \frac{M(1 - \exp(-(p+q)\,T))}{1 + \exp(-\left(\frac{q}{p}\right)(p+q)T)}$$

where V = Sales of innovative agro-industrial product

M = Maximum sales of innovative agro-industrial product

p = Coefficient of innovation

q = Coefficient of imitation

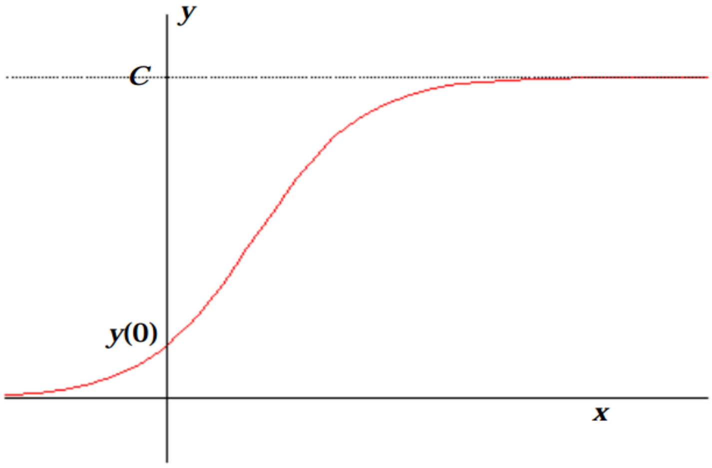
T = Time

2.2 Logistic function

For real numbers *a, b,* and *c*, the function

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

is a Logistic function. If $a>0$, a logistic function increases when $b>0$ and decreases when $b<0$. The number, c, is called the *limiting value* or the *upper limit* of the function because the graph of a logistic growth function will have a horizontal asymptote at $y = c$.



As is clear from the graph above, the characteristic S-shape in the graph of a Logistic function shows that initial exponential growth is followed by a period in which growth slows and then levels off, approaching (but never attaining) a maximum upper limit.

Stoneman (2010) suggested the Logistic function for the forecasting of sales of new products especially soft innovation as follows:

$$V = \frac{M}{1 + \mathrm{A} * \exp\left(-\beta T\right)}$$

where V = Sales of innovative agro-industrial product

M = Maximum sales of innovative agro-industrial product

β = Parameter

T = Time

3. Methodology

The methodology to estimate parameters in Bass model and the Logistic function form can be proceed in 4 ways as follows:

Method 1: Least squares using quadratic interpolation algorithm

The parameter estimation includes these following steps.

Step 1: Initiate three initial values of parameter M. Transform the data using logistic transformation into linear function.

$$ln\left(\frac{V/M}{1 - V/M}\right) - \ln\left(\frac{1}{\mathrm{A}}\right) = \beta \mathrm{T}$$

Then, estimate parameter β using Ordinary Least Squares (OLS)

Step 2: Take parameter M and β to forecast sales by this formula.

$$\hat{V} = \frac{M}{1 + A * \exp(-\beta T)}$$

The value of A will be calculated by this formula to fix the y-intercept at the first data of the series (Vo).

$$A = \frac{M}{V} - 1$$

Step 3: Calculate the Sum Squared Error (SSE).

$$\sum e^2 = \sum_{i=1}^{N} (V_i - \widehat{V_i})$$

Step 4: Calculate the SSE at the three points using the three initial M values.

Step 5: Search for a new M value by using Quadratic Interpolation

Step 6: Include the new M with other two previous M values which are located nearest to the new M. Then, estimate parameter β and calculate the SSE again.

Step 7: Repeat step 5 and 6 for 10,000 iterations.

Step 8: Summarize the values of parameter M and β.

Method 2: Least squares using Quasi-Newton algorithm

The parameter estimation includes these following steps.

Step 1: Repeat step 1 to 4 of method 1 (Least squares using quadratic interpolation algorithm). This will yield the values of M, β and SSE. Each parameter will contain three values.

Step 2: Calculate the slope between the values of M, β and SSE. Two slopes will be available for each parameter.

Step 3: Initiate the initial value of H (Ho). It should be the identity matrix at the size of 2 ×2.

Step 4: Calculate a new H using this formula.

$$H = H_o + \frac{vv'}{v'u} - \frac{H_o u u' H_o}{u' H_o u}$$

where v = Difference of the parameter

u = Difference of the slope of the parameter

Step 5: Calculate the increment of the parameter by this formula.

$$d = -Hg$$

where d = The increment of the parameter

g = Initial slope of the parameter

Step 6: Calculate a new parameter by adding the increment to the previous parameter.

Step 7: Create two nearby values for parameter M. Repeat the process for parameter β.

Step 8: Calculate the SSE from the new parameter M and β.

Step 9: Repeat step 4 to 8 for 10,000 iterations.

Step 10: Summarize the values of parameter M and β.

Method 3: Maximum likelihood using quadratic interpolation algorithm

This method is like the least squares using quadratic interpolation algorithm. It changes the objective function to be the likelihood function as follows:

$$L = \prod_{i=1}^{T} Pr(V_i|T_i)$$

and

$$Pr(V_i|T_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\left(-\frac{1}{2}\right)\frac{(Vi - Fi)^2}{\sigma^2}\right\}$$

where Pr(Vi|Ti) = Probability of the occurrence of a sales value at a time

σ = Variance

Vi = Sales value

Fi = Forecasted sales value

Method 4: Maximum likelihood using Quasi-Newton algorithm

This method is quite similar to method 3 (Maximum likelihood using quadratic interpolation algorithm). It changes the objective function to be the likelihood function as follows:

$$L = \prod_{i=1}^{T} Pr(V_i|T_i)$$

and

$$Pr(V_i|T_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\left(-\frac{1}{2}\right)\frac{(Vi - Fi)^2}{\sigma^2}\right\}$$

The details of the equations are described in method 3.

To compare the performance between Bass model and logistic function. The study will calculate the Mean Absolute Percentage Error (MAPE) in the out-of-sample test. Then match the models which are estimated by the same method and compare their MAPEs. Moreover, It will compare the MAPE of the best Bass model to the best logistic function. Statistics that will be used to test the hypothesis is t-statistics.

4. Data

Data are from the Royal Project. They are monthly data ranged from January 2010 to August 2012. Totally, the model has 32 observations.

5. Results

The results show the comparison between Bass model and logistic function for the whole observations (32 observations from January 2010 to August 2012) and for the selected observations (from observation 7 to 24).

5.1 Logistic function

5.1.1 Logistic 1

The estimation result of Logistic function using maximum likelihood with quadratic interpolation (to search for M) with fixed intercept at Vo (Logistic 1) is presented in table 1.

TABLE 1. Estimation result of Logistic function using maximum likelihood with quadratic interpolation (to search for M) with fixed intercept at Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.15E+06	0.2049	710.7455
4	1.15E+06	0.1352	341.249
5	1.15E+06	0.1257	299.5281
6	1.15E+06	0.099	172.7165
7	1.15E+06	0.042	26.9965
8	1.15E+06	4.77E-02	30.1830
9	1.17E+06	0.0657	60.8877
10	1.15E+06	0.0486	30.1596
11	1.15E+06	4.14E-02	27.0777
12	1.15E+06	0.0432	28.0433
13	1.15E+06	3.65E-02	29.7533
14	2.69E+06	0.0482	30.7983
15	2.20E+06	0.0517	36.7008
16	2.20E+06	0.0511	37.3895
17	2.19E+06	0.0495	36.282
18	4.05E+06	0.0423	25.6119
19	5.00E+06	0.0458	27.9018
20	4.11E+06	0.0482	31.9772
21	5.03E+06	0.047	31.4440
22	3.40E+06	0.0488	35.5675
23	1.94E+06	0.05	39.7354
24	5.25E+06	0.0487	42.5036
25	3.12E+06	0.0508	50.3024
26	1.40E+07	0.0473	35.0448
27	2.10E+07	0.0465	36.2831
28	2.10E+07	0.0463	43.6599
29	2.57E+07	0.0458	52.2262
30	2.14E+07	0.0462	75.3084
31	4.67E+07	0.0449	96.1237
32	6.60E+07	0.0429	-

Source: Own calculation

5.1.2 Logistic 2

The estimation result of Logistic function using least squares with quadratic interpolation and fixed intercept at Vo (Logistic 2) is presented in table 2.

TABLE 2. Estimation result of Logistic function using least squares with quadratic interpolation (to search for M) and fixed intercept at Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.80E+06	0.2028	936.1434
4	1.48E+06	0.1343	371.2554
5	1.48E+06	0.1248	321.8111
6	1.80E+06	9.80E-02	185.2782
7	1.15E+06	0.042	26.9965
8	1.15E+06	0.0477	3.02E+01
9	1.17E+06	0.0657	60.8896
10	1.15E+06	0.0486	30.1602
11	1.15E+06	0.0414	27.0777
12	1.15E+06	0.0432	28.0434
13	1.15E+06	0.0365	2.98E+01
14	1.15E+06	4.92E-02	30.4141
15	1.16E+06	0.0527	36.0846
16	1.16E+06	0.052	36.7475
17	1.15E+06	0.0503	35.8103
18	1.47E+06	0.0431	25.4413
19	1.48E+06	0.0468	27.5098
20	1.16E+06	0.0496	31.3205
21	1.48E+06	0.048	30.9512
22	1.17E+06	0.0501	34.8615
23	1.25E+06	0.0507	39.3213
24	1.19E+06	0.0502	41.5403
25	1.10E+06	0.0523	48.9638
26	1.81E+06	0.0484	34.4178
27	1.81E+06	0.0477	35.6350
28	1.81E+06	0.0475	42.9912
29	2.23E+06	0.0468	51.6853
30	1.84E+06	0.0474	74.1752
31	3.41E+06	0.0455	95.4737
32	4.16E+06	0.0434	-

Source: Own calculation

5.1.3 Logistic 3

The estimation result of Logistic function using maximum likelihood with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo (Logistic 3) is presented in table 3.

TABLE 3. Estimation result of Logistic function using maximum likelihood with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo

N	M*	Beta	Likelihood	Out-sample test (MAPE)
3	1.88E+06	0.2123	2.28E-15	1.04E+03
4	2.11E+06	0.1573	2.96E-20	622.3783
5	1.80E+06	0.1244	3.93E-25	338.1848
6	1.90E+06	0.1033	5.06E-30	215.7572
7	1.80E+06	0.0413	5.45E-35	27.0290
8	1.79E+06	4.68E-02	7.22E-40	30.1028
9	1.82E+06	0.0638	0.0638	59.5629
10	1.79E+06	0.0475	9.03E-50	29.9117
11	1.79E+06	0.0405	1.13E-54	27.1930
12	1.79E+06	0.0422	1.50E-59	28.0026
13	1.79E+06	3.58E-02	1.83E-64	29.8949
14	1.77E+06	0.0478	8.92E-70	29.7504
15	1.79E+06	0.051	1.15E-74	34.8033
16	1.79E+06	0.0504	1.50E-79	35.7265
17	1.78E+06	0.0488	1.88E-84	34.9676
18	1.78E+06	0.0424	1.48E-89	25.3802
19	1.78E+06	0.0458	1.40E-94	26.6273
20	1.79E+06	0.0479	1.62E-99	30.0215
21	1.78E+06	0.0469	1.96E-104	29.6755
22	1.82E+06	0.0483	2.28E-109	33.0818
23	1.90E+06	0.0487	3.00E-114	36.4223
24	1.85E+06	0.0483	3.77E-119	39.1461
25	1.68E+06	0.0504	2.90E-124	46.0518
26	1.78E+06	0.0475	1.23E-129	31.8652
27	1.78E+06	0.0468	1.42E-134	33.4098
28	1.78E+06	0.0467	1.86E-139	40.7572
29	1.78E+06	0.0462	2.26E-144	49.6259
30	1.81E+06	4.64E-02	2.86E-149	6.93E+01
31	1.78E+06	0.0455	2.09E-154	91.3084
32	1.74E+06	0.044	7.32E-160	-

Source: Own calculation

5.1.4 Logistic 4

The estimation result of Logistic function using least squares with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo (Logistic 4) is presented in table 4.

TABLE 4. Estimation result of Logistic function using least squares with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo

N	M*	Beta	SSE	Out-sample test (MAPE)
3	1.77E+06	0.2006	3.14E+12	9.09E+02
4	1.77E+06	0.1325	3.14E+12	3.81E+02
5	1.78E+06	0.1227	3.16E+12	3.25E+02
6	1.77E+06	0.0968	3.16E+12	1.79E+02
7	1.78E+06	0.0408	3.17E+12	2.71E+01
8	1.78E+06	0.0464	3.17E+12	2.98E+01
9	1.82E+06	0.0637	3.31E+12	59.2146
10	1.78E+06	0.0473	3.18E+12	2.97E+01
11	1.77E+06	0.0403	3.15E+12	27.2792
12	1.77E+06	0.0421	3.16E+12	2.80E+01
13	1.77E+06	0.0357	3.14E+12	2.99E+01
14	1.76E+06	0.048	3.13E+12	2.99E+01
15	1.79E+06	0.051	3.20E+12	3.47E+01
16	1.79E+06	0.0503	3.20E+12	35.6878
17	1.78E+06	0.0489	3.18E+12	3.50E+01
18	1.75E+06	0.0429	3.13E+12	2.55E+01
19	1.77E+06	0.046	3.15E+12	2.68E+01
20	1.79E+06	0.0479	3.22E+12	3.00E+01
21	1.78E+06	0.0469	3.19E+12	2.98E+01
22	1.82E+06	0.0483	3.31E+12	33.0224
23	1.90E+06	0.0486	3.63E+12	36.3320
24	1.85E+06	0.0483	3.43E+12	39.0104
25	1.68E+06	0.0504	2.81E+12	4.60E+01
26	1.79E+06	0.0476	3.22E+12	3.22E+01
27	1.79E+06	0.047	3.23E+12	33.7473
28	1.80E+06	0.0467	3.26E+12	4.09E+01
29	1.80E+06	0.0462	3.28E+12	49.5553
30	1.83E+06	0.0463	3.38E+12	6.87E+01
31	1.81E+06	0.0454	3.31E+12	9.05E+01
32	1.78E+06	0.044	3.24E+12	-

Source: Own calculation

5.1.5 Bass1

The estimation result of Bass model using least squares and searching for only M (fixed p and fixed q) with quadratic interpolation (Bass 1) is presented in table 5.

TABLE5. Estimation result of Bass model using least squares and searching for only M (fixed p and fixed q) with quadratic interpolation

N	M*	p*	q*	SSE	MAPE
3	7.92E+04	0.03	0.38	9.62E+08	56.6884
4	6.37E+04	0.03	0.38	1.15E+09	4.22E+01
5	5.92E+04	0.03	0.38	1.20E+09	40.2503
6	5.39E+04	0.03	0.38	1.34E+09	38.4486
7	4.57E+04	0.03	0.38	1.94E+09	36.4375
8	4.50E+04	0.03	0.38	1.95E+09	38.0884
9	4.84E+04	0.03	0.38	2.20E+09	36.9074
10	4.53E+04	0.03	0.38	2.50E+09	37.5299
11	4.36E+04	0.03	0.38	2.61E+09	39.2315
12	4.39E+04	0.03	0.38	2.62E+09	40.6573
13	4.26E+04	0.03	0.38	2.75E+09	42.2694
14	4.75E+04	0.03	0.38	4.91E+09	36.1523
15	4.93E+04	0.03	0.38	5.25E+09	34.6348
16	4.98E+04	0.03	0.38	5.27E+09	35.6607
17	4.99E+04	0.03	0.38	5.28E+09	37.5858
18	4.86E+04	0.03	0.38	5.62E+09	37.5954
19	5.11E+04	0.03	0.38	7.00E+09	34.1257
20	5.33E+04	0.03	0.38	8.09E+09	31.5217
21	5.37E+04	0.03	0.38	8.13E+09	33.0588
22	5.58E+04	0.03	0.38	9.53E+09	30.4537
23	5.75E+04	0.03	0.38	1.05E+10	29.04
24	5.84E+04	0.03	0.38	1.08E+10	29.4457
25	6.15E+04	0.03	0.38	1.47E+10	25.301
26	6.09E+04	0.03	0.38	1.48E+10	25.9967
27	6.17E+04	0.03	0.38	1.51E+10	26.5434
28	6.29E+04	0.03	0.38	1.58E+10	24.7814
29	6.38E+04	0.03	0.38	1.64E+10	23.8492
30	6.57E+04	0.03	0.38	1.87E+10	14.5013
31	6.60E+04	0.03	0.38	1.87E+10	19.2648
32	6.56E+04	0.03	0.38	1.88E+10	-

Source: Own calculation

5.1.6 Bass2

The estimation result of Bass model using least squares to search for M and q (fixed p) with Quasi-Newton (Bass 2) is presented in table 6.

TABLE 6. Estimation result of Bass model using least squares searching for M and q (fixed p) with Quasi-Newton

N	M*	p*	q*	SSE	MAPE
3	1.65E+05	0.03	0.1613	1.75E+10	170
4	1.32E+05	0.03	1.46E-01	3.34E+10	1.15E+02
5	1.11E+05	0.03	0.1335	4.57E+10	79.8921
6	9.10E+04	0.03	0.1234	5.64E+10	49.7268
7	7.28E+04	0.03	0.1098	6.28E+10	31.8864
8	6.27E+04	0.03	0.104	6.38E+10	3.04E+01
9	5.94E+04	0.03	0.1032	5.97E+10	30.2363
10	5.24E+04	0.03	9.77E-02	5.66E+10	33.2553
11	4.42E+04	0.03	9.74E-02	5.68E+10	4.15E+01
12	4.20E+04	0.03	0.0983	5.16E+10	44.0503
13	3.92E+04	0.03	9.80E-02	4.72E+10	4.91E+01
14	4.20E+04	0.03	0.1068	4.32E+10	44.7081
15	4.27E+04	0.03	0.1119	3.89E+10	4.37E+01
16	4.45E+04	0.03	0.1095	3.28E+10	42.7652
17	4.24E+04	0.03	0.1189	3.15E+10	45.8242
18	4.10E+04	0.03	1.20E-01	2.92E+10	4.88E+01
19	4.30E+04	0.03	0.1266	2.72E+10	45.5426
20	4.48E+04	0.03	0.1319	2.53E+10	42.6395
21	4.51E+04	0.03	0.135	2.32E+10	43.4889
22	4.69E+04	0.03	0.1396	2.21E+10	40.1709
23	4.83E+04	0.03	0.1435	2.10E+10	37.3793
24	4.92E+04	0.03	0.1461	1.96E+10	36.3505
25	5.43E+04	0.03	0.1285	1.91E+10	28.2265
26	4.85E+04	0.03	0.1982	2.39E+10	38.5536
27	4.82E+04	0.03	0.2294	2.41E+10	38.9902
28	5.03E+04	0.03	0.2098	2.26E+10	34.4547
29	5.12E+04	0.03	0.2152	2.21E+10	30.7847
30	5.28E+04	0.03	0.2213	2.34E+10	16.2434
31	5.32E+04	0.03	0.2258	2.28E+10	3.9243
32	5.30E+04	0.03	0.2297	2.23E+10	-

Source: Own calculation

5.1.7 Bass3

The estimation result of Bass model using least squares to search for M, p and q with Quasi-Newton (Bass 3) is presented in table 7.

TABLE 7.Estimation result of Bass model using least squares to search for M, p and q with Quasi-Newton

N	M*	p*	q*	SSE	MAPE
3	8.42E+05	-5.40E-02	6.43E-01	3.39E+09	1.00E+02
4	4.50E+05	-1.49E-02	3.59E-01	4.93E+09	1.00E+02
5	5.98E+05	-3.42E-02	4.84E-01	6.60E+09	1.00E+02
6	4.06E+05	-1.57E-02	3.48E-01	8.13E+09	1.00E+02
7	NaN	NaN	NaN	8.58E+09	NaN
8	2.64E+06	-3.04E-01	2.90E+00	9.80E+09	1.00E+02
9	-6.66E+05	0.1408	-0.7207	1.21E+10	6.19E+09
10	9.15E+04	0.0166	0.0709	1.31E+11	3.12E+01
11	1.11E+06	-0.1585	1.5482	1.55E+10	100
12	7.51E+04	0.0363	0.0597	1.34E+11	27.6333
13	2.80E+05	-0.0235	0.3995	1.88E+10	100
14	8.18E+04	0.0305	0.0592	1.00E+11	24.9463
15	1.02E+05	0.0981	0.0201	3.05E+10	27.4695
16	6.48E+04	0.0071	0.0812	1.19E+11	30.9202
17	4.55E+04	0.0361	4.16E-02	1.56E+11	52.6333
18	8.41E+04	0.1067	0.0275	4.77E+10	31.1679
19	NaN	NaN	NaN	4.59E+10	NaN
20	1.02E+05	0.1795	-1.68E-02	1.03E+10	47.2168
21	4.27E+04	0.0227	0.0597	1.51E+11	52.368
22	4.00E+04	0.0215	0.0411	1.61E+11	59.9345
23	1.12E+05	0.2081	-0.0195	1.16E+10	44.1433
24	3.97E+04	0.0205	0.0553	1.63E+11	54.8113
25	4.21E+04	0.0226	0.0612	1.63E+11	47.9314
26	8.82E+04	0.188	0.0222	3.21E+10	27.9643
27	1.02E+05	0.1989	0.003	1.57E+10	32.8234
28	-1.24E+05	-0.4592	0.1857	7.17E+10	5.54E+05
29	6.39E+04	0.1379	0.0408	9.27E+10	28.3513
30	7.74E+04	0.1557	0.0363	6.05E+10	14.0515
31	7.67E+04	0.156	0.0367	6.10E+10	11.049
32	9.27E+04	0.0945	0.0275	2.64E+10	-

Source: Own calculation

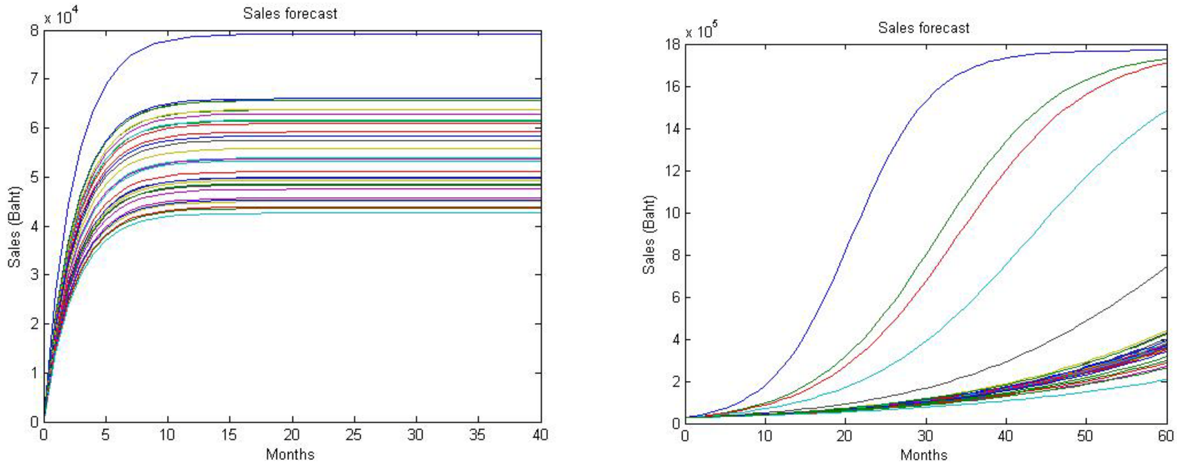


Figure 1. Forecasting results of Bass1 (the best of Bass model—on the left) and Logistic 4 (the best of Logistic function—on the right) show the maximum sales, growth of the sales and duration that the sales will reach the maturity period. Logistic function presents a clearer S-curve than Bass model.

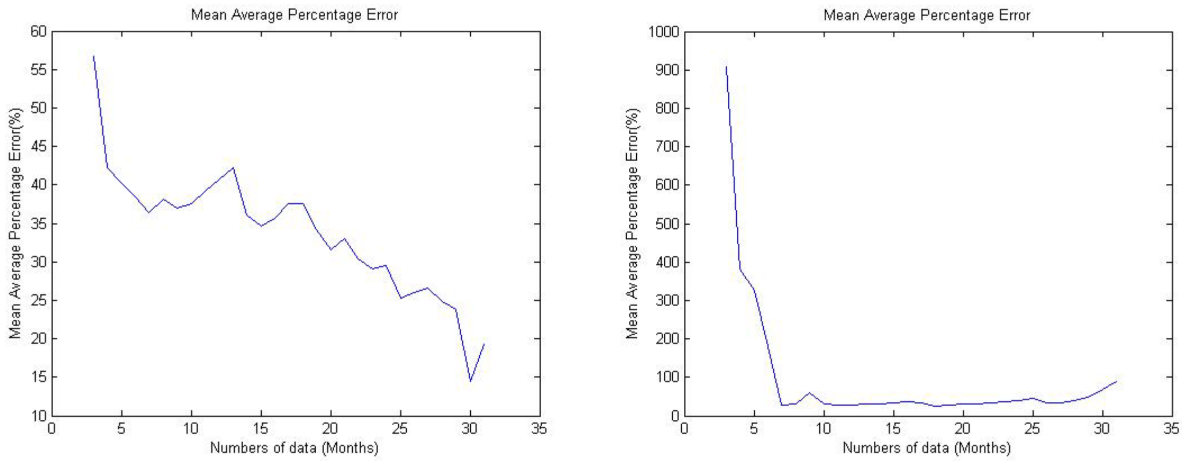


Figure 2. Mean Absolute Percentage Error (MAPE) of Bass1 (the best of Bass model on the left) and Logistic 4 (the best of Logistic function—on the right) at different numbers of observation. The MAPE of the Logistic function drops sharply at the 7th month.

5.2 Comparison forecasting results between Bass model and logistic function

5.2.1 Comparison for whole period

TABLE 1. Paired Samples Statistics using data from whole period

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic1	86.9724	29	142.45096	26.45248
	BASS1	33.7319	29	8.24126	1.53036
Pair 2	Logistic2	96.5840	29	181.94390	33.78613
	BASS2	46.1194	29	30.38460	5.64228
Pair 3	Logistic3	109.1484	29	217.52103	40.39264
	BASS2	46.1194	29	30.38460	5.64228
Pair 4	Logistic3	120.4531	25	232.82207	46.56441
	BASS3	53.8645	25	31.67147	6.33429
Pair 5	Logistic4	94.5568	29	178.35319	33.11936
	BASS2	46.1194	29	30.38460	5.64228
Pair 6	Logistic4	103.5253	25	190.98395	38.19679
	BASS3	53.8645	25	31.67147	6.33429

TABLE 1. (cont.)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95%Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic1-BASS1	53.2405	138.04961	25.63517	.7292	105.7518
Pair 2	Logistic2-BASS2	50.4646	154.90783	28.76566	-8.4592	109.3884
Pair 3	Logistic3-BASS2	63.0290	189.66798	35.22046	-9.1169	135.1748
Pair 4	Logistic3-BASS3	66.5886	218.13405	43.62681	-23.4527	156.6299
Pair 5	Logistic4-BASS2	48.4374	151.14980	28.06781	-9.0570	105.9317
Pair 6	Logistic4-BASS3	49.6608	177.40011	35.48002	-23.5664	122.8880

TABLE 1. (cont.)

		t	df	Sig. (2-tailed)
Pair 1	Logistic1-BASS1	2.077	28	.047
Pair 2	Logistic2- BASS2	1.754	28	.090
Pair 3	Logistic3- BASS2	1.790	28	.084
Pair 4	Logistic3- BASS3	1.526	24	.140
Pair 5	Logistic4- BASS2	1.726	28	.095
Pair 6	Logistic4- BASS3	1.400	24	.174

Source: Own calculation using SPSS

By the usage of the whole observations, Bass model is superior than logistic function. However, when we concern just for the selected period (7 to 24 months) which MAPE of logistic function are improved sharply, then we will compare the models again. The results are presented in the next section.

5.2.2 Comparison for selected period

We compare the MAPE of Bass model and logistic function just for the range of 7 to 24 months. The results are as follows:

TABLE 2. Paired Samples Statistics using data from 7th to 24th month

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic1	33.8341	18	8.26877	1.94897
	BASS1	35.5776	18	3.73683	.88078
Pair 2	Logistic2	33.5059	18	8.20418	1.93374
	BASS2	40.6569	18	6.02785	1.42078
Pair 3	Logistic3	32.6277	18	7.73921	1.82415
	BASS2	40.6569	18	6.02785	1.42078
Pair 4	Logistic3	31.6053	15	3.87010	.99926
	BASS3	52.2960	15	27.13091	7.00517
Pair 5	Logistic4	32.6051	18	7.63939	1.80062
	BASS2	40.6569	18	6.02785	1.42078
Pair 6	Logistic4	31.5824	15	3.82513	.98764
	BASS3	52.2960	15	27.13091	7.00517

TABLE 2. (cont.)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic1 BASS1	-1.7435	10.06917	2.37333	-6.7508	3.2638
Pair 2	Logistic2 BASS2	-7.1510	12.14158	2.86180	-13.1888	-1.1131
Pair 3	Logistic3 BASS2	-8.0291	11.76284	2.77253	-13.8786	-2.1796
Pair 4	Logistic3 BASS3	-20.6907	27.98427	7.22551	-36.1879	-5.1935
Pair 5	Logistic4 BASS2	-8.0518	11.63725	2.74293	-13.8388	-2.2647
Pair 6	Logistic4 BASS3	-20.7136	27.99801	7.22906	-36.2184	-5.2089

TABLE 2. (cont.)

		t	df	Sig. (2-tailed)
Pair 1	Logistic1-BASS1	-.735	17	.473
Pair 2	Logistic2- BASS2	-2.499	17	.023
Pair 3	Logistic3- BASS2	-2.896	17	.010
Pair 4	Logistic3- BASS3	-2.864	14	.013
Pair 5	Logistic4- BASS2	-2.935	17	.009
Pair 6	Logistic4- BASS3	-2.865	14	.012

Source: Own calculation using SPSS

By the usage of only selected period (7 to 24 months), logistic function is superior than Bass model . In 5 pairs out of 6. the MAPE of logistic function is significantly smaller than that of Bass model.

5.2.3 Comparison between the best of Bass model and logistic function

In this section, the best Bass model which is BASS1 and the best logistic function which is Logistic4 will be compared together. The results are shown as follows:

TABLE 3. Paired samples statistics between the best logistic model and the best Bass model

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic4	32.6051	18	7.63939	1.80062
	BASS1	35.5776	18	3.73683	.88078

TABLE 3. (cont.)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic4 BASS1	-2.9725	9.14291	2.15500	-7.5191	1.5742

TABLE 3. (cont.)

		t	df	Sig. (2-tailed)
Pair 1	Logistic4 BASS1	-1.379	17	.186

Source: Own calculation using SPSS

It is insignificantly different between the best Bass model and logistic function although the MAPE of the logistic function is a little bit lower than that of Bass model.

6. Conclusions

In conclusion, Logistic function is superior than Bass model when the model uses the data between 7 to 24 months where the MAPE of the Logistic function perform much better than other period of time. However, the best Logistic function is insignificantly superior than the best Bass model. Therefore, it can be said that Logistic function at least yield as good performance as Bass model.

REFERENCES

Bass, Frank. 1969. A New Product Growth Model for Consumer Durables. **Management Science 15** (January): pp. 215 – 227.

Mahajan, Muller and Frank Bass. 1990. New Product Diffusion Models in Marketing: A Review and Directions for Research. **Journal of Marketing 54** (January): 1 - 26.

Meade, Nigel and Towhidul Islam. 2006. Modelling and Forcasting the diffusion of innovation- A 25-year review. **International Journal of Forecating 22**: pp. 519 – 545.

Stoneman, Paul. 2010. **Soft Innovation: Economics, Product Aesthetics and the Creative Industries**. Oxford: Oxford University Press.

Rogers, Everett M. 2003. **Diffusion of Innovations**. 5th ed. New York: Free Press.