

## **ARFIMA-FIGARCH and ARFIMA -FIAPARCH on Thailand Volatility Index**

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### ARTICLE INFO

Keywords:  
Volatility index  
Model selection  
Fractional integrated  
Price forecasting  
Time series

JEL classification codes:  
C22; C53; G17

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### ABSTRACT

This paper applied SET50 Index options with the Chicago Board Options Exchange (CBOE) as a Thailand Volatility Index (TVIX). This can be considered as a hedging diversification tool because of the high negative correlation with stock index. In addition, we estimate ARFIMA-FIGARCH and ARFIMA-FIAPARCH which are capable of capturing long memory and asymmetry in the conditional variance and power transformed conditional variance of process. The empirical shows that the best model with accuracy is ARMA-FIAPARCH.

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### 1. Introduction

Volatility Index (VIX) measures market expectation of near term volatility conveyed by stock index option prices. The original VIX was constructed using the implied volatilities of eight different S&P100 (OEX) option series so that, at any given time, it represented the implied volatility of a hypothetical at-the-money OEX option with exactly 30 days to expiration from an option-pricing model.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which quickly became the benchmark for stock market volatility. In 2003, the CBOE made two key enhancements to the VIX methodology. The new VIX is based on an up-to-the-minute market estimation of expected volatility that is calculated by using real-time S&P 500

Index (SPX) option bid/ask quotes. Until 2006, VIX was trading on the CBOE. The VIX options contract is the first product on market volatility to be listed on an SEC-regulated securities exchange. This new product can be traded from an options-approved securities account. Many investors consider the VIX to be the world's premier barometer of investor sentiment and market volatility, and VIX option is a very powerful risk management tool.

For the econometric model, it is assumed to be *ceteris paribus* with the variance and error term as constant terms. ARCH (Autoregressive Conditional Heteroscedastic) was developed and applied to ARMA (Autoregressive Moving Average) model in order to correct the assumption contradiction of time series economics data. The data has high variance with non-stationary variance and error term (Enders, 1995). The investors simply focus on the conditional variance such as the

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prediction of return and variance. ARMA shows the value of mean and variance simultaneously (Engle, 1982). GARCH (Generalized Autoregressive Conditional Heteroscedastic) model was developed from ARCH in order to adjust the variance to characterize as ARMA process which is applied in time variance model in money market (Engle, 1982 and Bollerslev, 1986).

Baillie, Bollerslev, and Mikkelsen (1996) proposed FIGARCH (Fractional Integrated GARCH) model which is effectively capture both volatility clustering and long memory because GARCH model exhibit short memory and cannot analyze hyperbolic memory in conditional volatility process and capture asymmetries in equity market volatility. With the previous findings of Ding, Granger, and Engle (1993) and Baillie *et al.* (1996) (among others) who suggest the modeling of conditional variance of high frequency financial data by the use of an (Asymmetric) Power GARCH (APARCH) or Fractionally Integrated GARCH (FIGARCH) models. Tse (1998) develops the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model, which allows for long memory and asymmetries in volatility.

This paper is to calculate TVIX basing on SET50 index options with CBOE VIX formula with the nearest-month contracts and compare the best performance of ARFIMA-FIGARCH and -FIAPARCH models for forecasting TVIX.

## 2. Volatility Index

Estimating implied volatility from options is no straightforward method to extract the information. Whaley (2000) considered implied volatility as a fear gauge because option prices calculate implied volatility that represents a market-based estimate of future price volatility). Implied volatilities are the information by investors, financial news services and other finance professionals. The information content and forecast quality of implied volatility is an

important topic in financial markets research.

Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981) and Jorion (1995) provided early assessments of the forecast quality of implied volatility and concluded that implied volatility outperforms historical standard deviations and is a good predictor of future volatility, although it might be biased. Christensen and Prabhala (1998) also found that implied volatility forecasts are biased, but dominate historical volatility in terms of ex ante forecasting power. Fleming (1998) used a historical volatility measure to show that implied volatilities outperform historical information.

Dennis *et al.* (2006) found that daily innovations in VIX contain very reliable incremental information about the future volatility of the S&P100 index. Other studies that attempt to forecast implied volatility or use the information contained in implied volatility to trade in option markets include Harvey and Whaley (1992), Noh *et al.* (1994), and Poon and Pope (2000).

## 3. New VIX Procedure

The New VIX is more robust because it pools the information from option prices over the whole volatility skew, and not just from at-the-money options. The formula used in the new VIX calculation is given by the CBOE as follows:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2$$

where

$\sigma$  = VIX / 100 (so that VIX =  $\sigma \times 100$ ),

T = Time to expiration (in minutes),

F = Forward index level, derived from index option prices (based on at-the-money option prices, the difference between call and put prices is smallest).

The formula used to calculate the forward index level is:

$$F = \text{Strike price (at-the-money)} + e^{RT} \times (\text{Call price} - \text{Put price}),$$

where

R = risk-free interest rate is assumed to be 3.01% (for simplicity, the government T-bills 3 month contract interest rate is used, as the Thailand options contract is a 3 months contract);

$$T = \{M_{\text{current day}} + M_{\text{settlement day}} + M_{\text{other days}}\} / \text{minutes in a year},$$

where

$M_{\text{current day}}$  = # of minutes remaining until midnight of the current day,

$M_{\text{settlement day}}$  = # of minutes from midnight until 9:45 am on the TFEX settlement day,

$M_{\text{other days}}$  = Total # of minutes in the days between the current day and the settlement day;

$K_i$  = Strike price of  $i$ th out-of-the-money option; a call if  $K_i > F$  and a put if  $K_i < F$ ;

$\Delta K_i$  = Interval between strike prices - half the distance between the strike on either side of  $K_i$ :

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}.$$

$K_0$  = First strike below the forward index level, F;

$Q(K_i)$  = The midpoint of the bid-ask spread for each option with strike  $K_i$ .

(Note:  $\Delta K_i$  for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise,  $\Delta K$  for the highest strike is the difference between the highest strike and the next lower strike.)

With the adaptation of the VIX calculation to Thailand SET 50 index options, the Thailand expected volatility (TVIX) can be estimated.

## 4. Theory

### 4.1 ARFIMA Model

ARIMA models are frequently used for seasonal time series (Box and Jenkins, 1976). A general multiplicative seasonal ARIMA model for time series  $Z_t$  is as follows:

$$\phi(L)\Phi(L^S)(1-L)^d(1-L^S)^D Z_t = \theta(L)\rho(L^S)a_t$$

where:

L = a backshift or lag operator  
( $B_{zt} - Z_{t-1}$ )

S = seasonal period

$$\phi(L) = (1 - \phi_1 L - \dots - \phi_p L^p)$$

is the non-seasonal AR operator

$$\Phi(L^S) = (1 - \Phi_1 L^S - \dots - \Phi_s L^{S^s})$$

is the seasonal AR operator

$$\theta(L) = (1 - \theta_1 L - \dots - \theta_q L^q)$$

is the non-seasonal MA operator

$$\rho(L) = (1 - \rho_1 L^S - \dots - \rho_0 L^{Q_s})$$

is the seasonal MA operator

$(1-L)^d(1-L^S)^D$  = non-seasonal differencing of order  $d$  and seasonal differencing of order  $D$

Granger and Joyeux (1980) and Hosking (1981) proposed an autoregressive fractionally integrated moving-average (ARFIMA) model and proposed the method to fit long-memory data. ARFIMA(p,d,q) is written as follow:

$$\phi(L)\Delta^d y_t = \delta + \theta(L)u_t$$

with:

$$\phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p \text{ and}$$

$$\theta(L) = 1 - \theta_1 L - \dots - \theta_q L^q$$

where:

- $\delta$  = a constant term
- $\theta(L)$  = the MA operator at order  $q$
- $u_t$  = an error term
- $\phi(L)$  = the AR operator at order  $p$
- $\Delta^d y_t$  = the differencing operator at

order  $d$  of time series data  $y_t$

For  $d = (-0.5, 0)$ , the process exhibits intermediate memory or long range negative dependence, while  $d = (0, 0.5)$ , the process exhibits long memory or long range positive dependence. For  $d = [0.5, 1)$ , the process is mean reverting with no long run impact to future values of the process and the process becomes a short memory when  $d = 0$  corresponding to a standard ARMA process.

### 4.2 FIGARCH Model

The GARCH model by Bollerslev (1986) imposed important limitations, not to capture a positive or negative sign of  $u_t$ , which both positive and negative shocks has the same impact on the conditional variance,  $h_t$ , as follows,

$$u_t = \eta_t \sqrt{\sigma_t},$$

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i u_{t-i}^2$$

where  $\omega > 0$ ,  $\alpha_i \geq 0$  for  $i = 1, \dots, p$  and  $\beta_j \geq 0$  for  $j = 1, \dots, q$  are sufficient to ensure that the conditional variance,  $h_t$  is non-negative. For the GARCH process to be defined, it is required that  $\omega > 0$ . Therefore, a univariate GARCH(1,1) model is known as ARCH( $\infty$ ) model (Engle, 1982) as an infinite expansion in  $u_{t-i}^2$ .

Baillie *et al.* (1996) proposed fractionally integrated GARCH (FIGARCH) model to determine long memory in return volatility. The FIGARCH (p,d,q) process is as follow:

$$\phi(L)(1-L)^d u_t^2 = \omega + [1 - \beta(L)]v_t,$$

where  $v_t = u_t^2 - \sigma_t^2$ ,  $0 < d < 1$ ,

$\phi(L) = \sum_{i=1}^{m-1} \phi_i L^i$  is of order  $m-1$ , and all the roots of  $\phi(L)$  and  $[1 - \beta(L)]$  lie outside the unit circle. The FIGARCH model is derived from standard GARCH model with fractional difference operator,  $(1-L)^d$ . The FIGARCH(p,d,q) model is reduced to the standard GARCH when  $d = 0$  and becomes IGARCH model when  $d = 1$ .

Baillie *et al.* (1996) claimed with the arguments of Nelson (1990) that the FIGARCH(p,d,m) is ergodic and strictly stationary which is difficult to verify. The degree of persistence of the FIGARCH model operates reversely direction of the ARFIMA process.

Chung (2001) suggested the analysis of the FIGARCH specification

$$\sigma_t^2 = \left\{ 1 - [1 - \beta(L)]^{-1} (1-L)^d \phi(L) \right\} \varepsilon_t^2$$

### 4.3 FIAPARCH Model

Tse (1998) extended the asymmetric power ARCH (APARCH) model of Ding *et al.* (1993) to fractionally integrated of Baillie *et al.* (1996) which is extended to FIAPARCH model as follows:

$$\sigma_t^\delta = \omega + \left[ 1 - \frac{[1 - \phi(L)](1-L)^d}{1 - \beta(L)} \right] \left[ |u_t| - \gamma u_t \right]^\delta$$

where  $0 < d < 1$ ,  $\omega, \delta > 0, \phi, \beta < 1$ ,  $-1 < \gamma < 1$  and  $L$  is the lag operator. When  $\gamma > 0$ , negative shocks have a higher volatility than positive shocks. The particular value of power term may lead to suboptimal modeling and forecasting performance. Ding *et al.* (1993) found that the closer of  $d$  value converge to 1, the larger the memory of the process becomes. The process of FIAPARCH allows for asymmetry. When  $\gamma = 0$  and  $\delta = 2$ , the process of FIAPARCH is reduced to FIGARCH process.

ARFIMA-FIAPARCH generates the long memory property in both the first and (power transformed) second conditional moments and is sufficiently flexible to handle the dual long memory behavior. It can recognize the long memory aspect and provides an empirical measure of real uncertainty that accounts for long memory in the power transformed conditional variance of the process.

## 5. Data Descriptive

One-minute intervals of SET50 Index options are obtained from Bloomberg accounted by the Faculty of Economics, Chiang Mai University and Research Institute, Stock Exchange of Thailand. The sample period is from 27 January 2008 until 30 September 2009. The contract months

are March, June, September 2008 and 2009 and December 2008.

In order to calculate TVIX, we use the SAS 9.1 software package for the calculation as it offers a number of features that are not available in traditional econometric software. Also, OxMetrics5 software is used to estimate ARFIMA-FIARCH and -FIAPARCH on daily returns.

The returns of TVIX at time  $t$  are calculated as follows:

$$R_{i,t} = \log(P_{i,t} / P_{i,t-1})$$

where  $P_{i,t}$  and  $P_{i,t-1}$  are the closing prices of TVIX at time  $t$  and  $t-1$ , respectively.

**Table 1: Descriptive Statistics of TVIX Returns**

	Mean	Std Dev	Skewness	Kurtosis	Max	Min
TVIX	-0.00022	0.093806	-1.08530	11.41235	0.44728	-0.52266

Table 1 presents the descriptive statistics for the returns of TVIX. The average return of TVIX is negative. The normal distribution has a skewness statistic equal to zero and a kurtosis statistic of 3, but return of TVIX has negative skewness statistics and high kurtosis, suggesting the presence of fat tails. This means that the data has a longer left tail (extreme losses) than right tail (extreme gain). Figure 1 presents the plot of TVIX and TVIX returns. This indicates some circumstances where TVIX returns fluctuate.

Table 2 summarized the unit root tests for TVIX returns. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. The tests yield large negative values in all cases for levels such that the individual returns series reject the null hypothesis at the 1% significance level, hence, the returns are stationary.

Figure 1: Daily TVIX and Returns

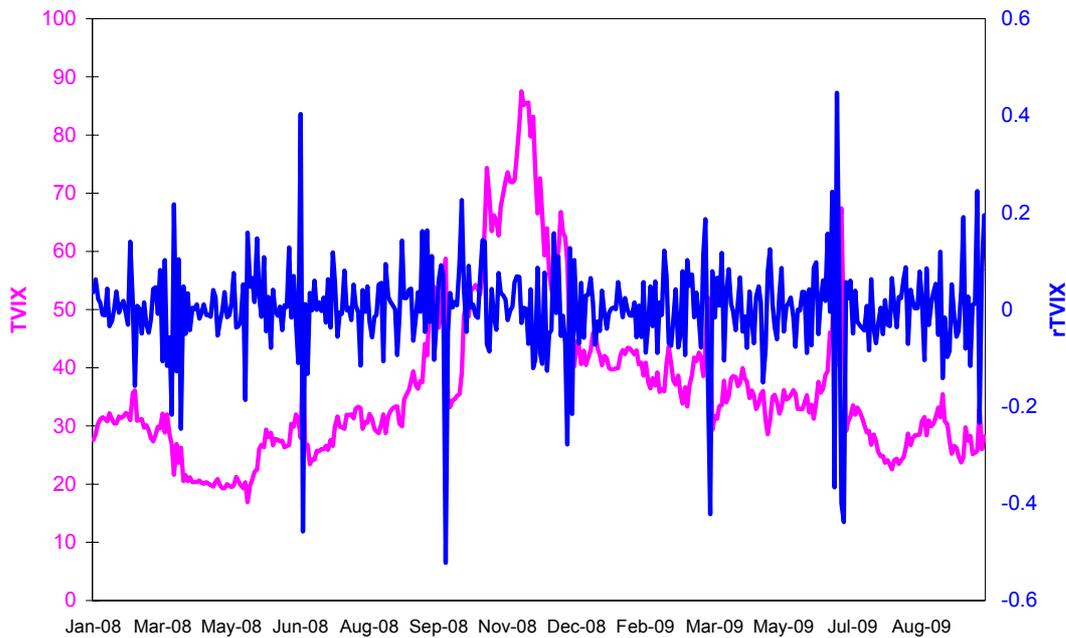


Table 2: Unit Root Test for Returns of TVIX

Returns	ADF Test			Phillips-Perron Test		
	None	Constant	Constant and Trend	None	Constant	Constant and Trend
TVIX	-24.022*	-23.991*	-23.977*	-24.560*	-24.526*	-24.522*

Note: \* significant at the 1% level.

6. Estimation

The Akaike Information Criterion (AIC) (Akaike, 1973) and Schwarz Bayesian Information Criterion (SBIC) (Schwarz, 1978) are useful to determine the best fit among several competing nested or non-nested models. The model with the lowest values of AIC and SBIC is selected as fitting the sample data better.

Table 3 shows forecasting method based on ARFIMA-FIGARCH and ARFIMA-FIAPARCH models for forecasting TVIX. The values of both AIC and SBIC in each of ARFIMA-FIGARCH and ARFIMA-FIAPARCH model are used for selection the best ARFIMA-FIGARCH and ARFIMA-FIAPARCH models for forecasting TVIX for this period.

The lowest values of AIC and SBIC are 3.882 and 35.466, respectively for ARFIMA(1,  $d_m$ , 1)-FIGARCH(1,  $d_v$ , 1) and 5.848 and 41.381, respectively for ARFIMA(1,  $d_m$ , 1)-FIAPARCH(1,  $d_v$ , 1).

For ARFIMA(1,  $d_m$ , 1)-FIGARCH(1,  $d_v$ , 1) and ARFIMA(1,  $d_m$ , 1)-FIAPARCH(1,  $d_v$ , 1),  $d_m$  for ARFIMA(1,  $d_m$ , 1) is -0.0857 and -0.0901, respectively. For ARFIMA(3,  $d_m$ , 3)-FIGARCH(1,  $d_v$ , 1) and ARFIMA(3,  $d_m$ , 3)-FIAPARCH(1,  $d_v$ , 1),  $d_m$  of ARFIMA is -0.0491 and -0.0475, respectively. All  $d_m$  of ARFIMA processes are not statistically significant. This can be concluded that the process is ARMA-FIGARCH and ARMA-FIAPARCH. Both processes are

intermediate memory or long range negative dependence for TVIX.

The estimations of both  $d_v$  for ARFIMA(1,  $d_m$ , 1)-FIGARCH(1,  $d_v$ , 1) and ARFIMA(3,  $d_m$ , 3)-FIGARCH(1,  $d_v$ , 1) are 0.5397 and 0.5568 which are more than 0.5 at 1% level of significance. This means that both processes are statistically significant long memory and can be estimated in long run. However, the estimations of both  $d_v$  for ARFIMA(1,  $d_m$ , 1)-FIAPARCH(1,  $d_v$ , 1) and ARFIMA(3,  $d_m$ , 3)-FIAPARCH(1,  $d_v$ , 1) are 0.3786 and 0.3950 which are less than 0.5 at 1% level of significance. This means that both processes are statistically significant short memory and cannot be estimated in long run. FIAPARCH is not long memory because  $d_v$  is less than 0.5.

Hence, ARFIMA(3,  $d_m$ , 3)-FIGARCH(1,  $d_v$ , 1) has the largest memory than others which can be estimated in long run.

From table 4, mean absolute error (MAE) and mean absolute percent error (MAPE) of all models show that ARFIMA(3,  $d_m$ , 3)-FIGARCH(1,  $d_v$ , 1) is fitted in forecasting returns of TVIX as the lowest of both values.

Consequently, with the lowest AIC and SBIC, ARFIMA(1, -0.0857, 1)-FIGARCH(1, 0.5397, 1) and ARFIMA(1, -0.0910, 1)-FIAPARCH(1, 0.3786, 1) models are fitted to the data. However, comparing both processes, ARFIMA(1, -0.0857, 1)-FIGARCH(1, 0.5397, 1) is better than ARFIMA(1, -0.0910, 1)-FIAPARCH(1, 0.3786, 1). ARFIMA(3, -0.0491, 3)-FIGARCH(1, 0.5568, 1) model provides the best fit to the data with the lowest of MAE and MAPE in forecasting and has the largest memory than others which can be estimated in long run.

However, the process of ARFIMA is not statistically significant. Therefore, the null hypothesis of ARFIMA is accepted. The process becomes a short memory corresponding to a standard ARMA process.

ARMA-FIGARCH and ARMA-FIAPARCH are estimated as follows:

From table 5, the AIC and SBIC criteria values strongly favor the ARMA-FIGARCH formulation over the ARMA-FIAPARCH. However, the value of  $d_v$  for ARMA-FIAPARCH is greater than FIGARCH with statistically significant at the 1% level. This means that the ARMA-FIAPARCH process is statistically significant longer memory and can be estimated in long run than ARMA-FIGARCH, also with the larger statistically significant value of power term. Moreover, positive shocks have a higher volatility than negative shocks as  $\gamma < 0$ . This implies that positive shocks on TVIX are negative shocks in index options because TVIX and SET50 Index options are oppositely correlated.

## 7. Conclusion

This paper applies SET50 Index options with the CBOE volatility index, VIX, formulae as a TVIX which is the benchmark for stock market volatility and leveraging, whereby leverage allows traders to make a significant amount of money from a relatively small change in price.

We also analyze ARFIMA-FIGARCH - FIAPARCH models for the best prediction for returns of TVIX. From the viewpoint of AIC and SBIC, ARFIMA(1,  $d_m$ , 1)-FIGARCH(1,  $d_v$ , 1) is the best fit for modeling. However, from the viewpoint of MAE, MAPE and  $d_v$ , our empirical results show that ARFIMA(3,  $d_m$ , 3)-FIAPARCH(1,  $d_v$ , 1) is the best accuracy in forecasting returns of TVIX with the largest memory.

Moreover, with the statistically significant of  $d_m$  and  $d_v$ , the process becomes ARMA-FIGARCH and ARMA-FIAPARCH for the data. Therefore, both processes are also estimated. The results show that ARMA-FIGARCH is better fit to the data by using AIC and SBIC criteria values, however, ARMA-FIAPARCH is

longer memory than ARMA-FIGARCH and capture asymmetric effect.

However, both the Stock Exchange of Thailand and Security Exchange Commission should firstly develop and

launch TVIX as a hedging diversification tool in the market in order, for the investors, to learn and be acquainted with TVIX for a few years primarily, and apply the forecasted model to forecast TVIX.

**Table 3: Accuracy comparison in sample for different forecasting models of ARFIMA-FIGARCH and ARFIMA-FIAPARCH based on concept of both AIC and SBIC criterion**

ARFIMA-FIGARCH	AIC	SBIC
ARFIMA(1,d,1)-FIGARCH(1,d,1) d of ARFIMA = -0.0857, d of FIGARCH = 0.5397* (0.2509) (0.0000)	3.882	35.466
ARFIMA(1,d,2)-FIGARCH(1,d,1) d of ARFIMA = -0.1367, d of FIGARCH = 0.5435* (0.4902) (0.0000)	5.883	41.416
ARFIMA(1,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0331, d of FIGARCH = 0.5336* (0.7817) (0.0000)	7.880	47.360
ARFIMA(2,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0998, d of FIGARCH = 0.5277* (0.6300) (0.0000)	9.877	53.306
ARFIMA(3,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0491, d of FIGARCH = 0.5568* (0.5356) (0.0000)	11.836	59.226
ARFIMA-FIAPARCH	AIC	SBIC
ARFIMA(1,d,1)-FIAPARCH(1,d,1) d of ARFIMA = -0.0901, d of FIAPARCH = 0.3786* (0.3152) (0.0001)	5.848	41.381
ARFIMA(1,d,2)-FIAPARCH(1,d,1) d of ARFIMA = -0.0858, d of FIAPARCH = 0.3797* (0.5508) (0.0002)	7.848	47.329
ARFIMA(2,d,3)-FIAPARCH(1,d,1) d of ARFIMA = -0.1388, d of FIAPARCH = 0.3694** (0.7469) (0.0203)	11.843	59.219
ARFIMA(3,d,3)-FIAPARCH(1,d,1) d of ARFIMA = -0.0475, d of FIAPARCH = 0.3940* (0.6683) (0.0005)	13.823	65.147

Note: \* and \*\* significant at the 1% and 5% level, respectively.

**Table 4: MAE and MAPE of rTVIX by ARFIMA-FIGARCH and ARFIMA-FIAPARCH**

<b>ARFIMA-FIGARCH</b>							
<b>Model</b>	<b>Day</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>Average</b>
ARFIMA(1,d,1)- FIGARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.67	0.95	0.99	0.94	1.00	1.11
ARFIMA(1,d,2)- FIGARCH(1,d,1)	MAE	0.01	0.24	0.23	0.08	0.19	0.15
	MAPE (%)	0.69	1.00	1.00	0.98	0.99	0.93
ARFIMA(1,d,3)- FIGARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.20	0.15
	MAPE (%)	1.58	0.95	0.98	0.95	1.01	1.09
ARFIMA(2,d,3)- FIGARCH(1,d,1)	MAE	0.02	0.24	0.23	0.08	0.20	0.15
	MAPE (%)	1.63	0.96	0.98	0.93	1.01	1.10
ARFIMA(3,d,3)- FIGARCH(1,d,1)	MAE	0.00	0.25	0.21	0.08	0.18	0.14
	MAPE (%)	0.22	1.01	0.90	0.98	0.91	0.80
<b>ARFIMA-APARCH</b>							
<b>Model</b>	<b>Day</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>Average</b>
ARFIMA(1,d,1)- FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.56	0.96	0.99	0.94	1.00	1.09
ARFIMA(1,d,2)- FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.56	0.96	0.99	0.93	1.00	1.09
ARFIMA(2,d,3)- FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.40	0.96	0.99	0.93	1.00	1.06
ARFIMA(3,d,3)- FIAPARCH(1,d,1)	MAE	0.00	0.24	0.21	0.08	0.18	0.15
	MAPE (%)	0.34	1.00	0.92	1.01	0.93	0.84

Figure 2.1: Actual and Forecasting rTVIX by ARFIMA(1,d,1)-FIGARCH(1,d,1)

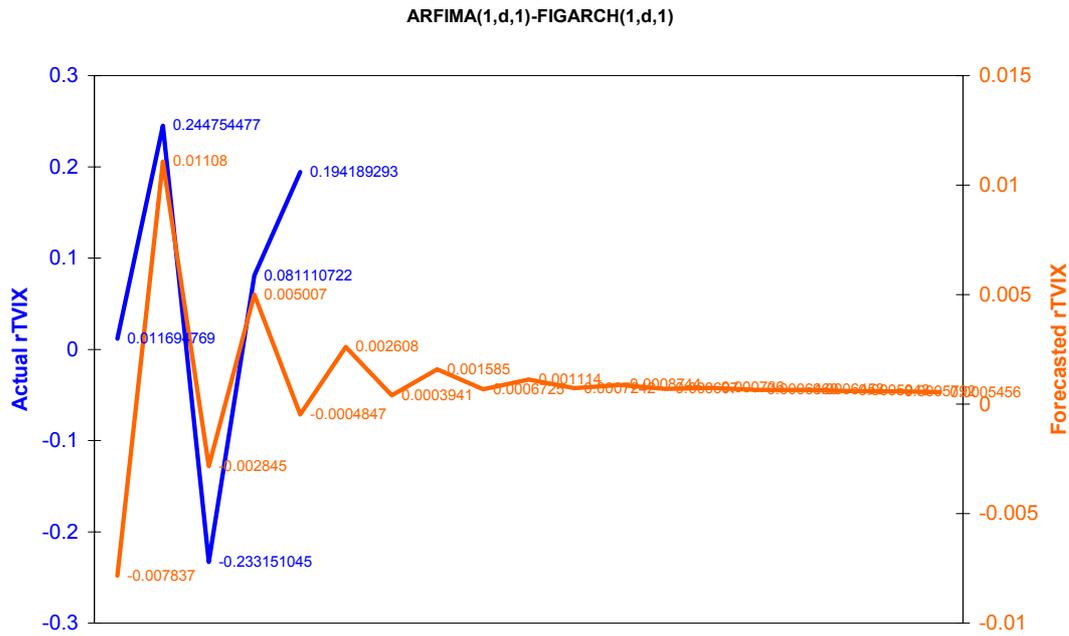
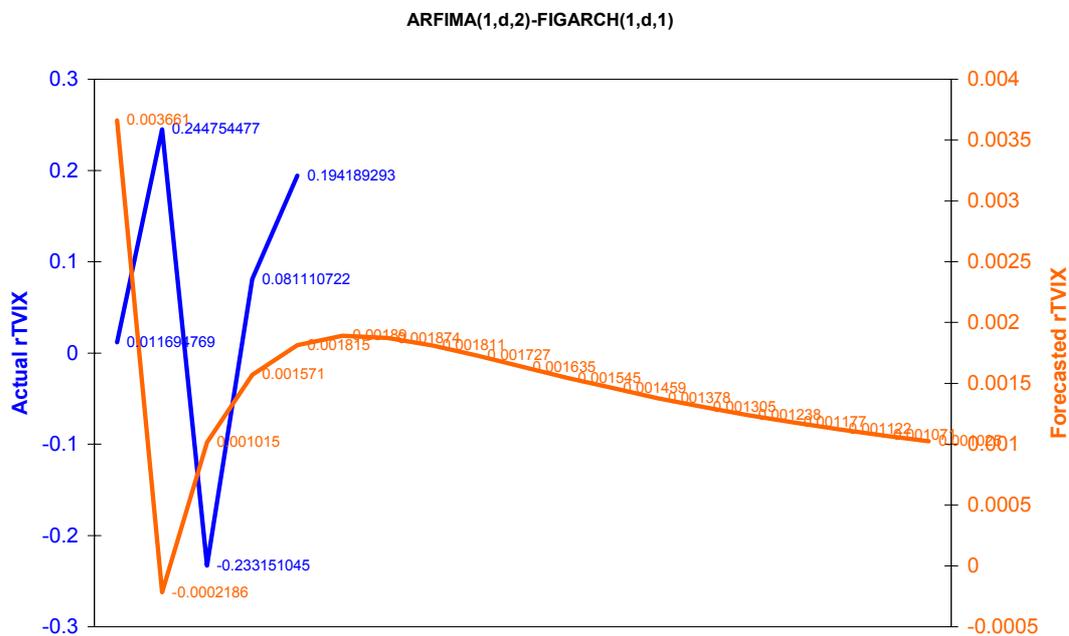
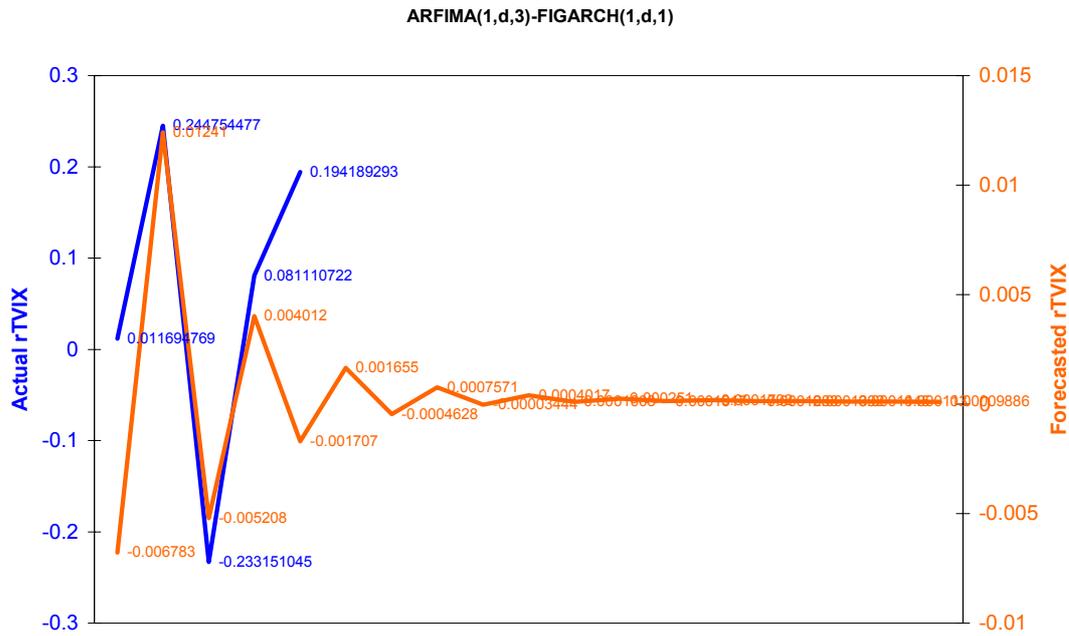


Figure 2.2: Actual and Forecasting rTVIX by ARFIMA(1,d,2)-FIGARCH(1,d,1)



**Figure 2.3: Actual and Forecasting rTVIX by ARFIMA(1,d,3)-FIGARCH(1,d,1)**



**Figure 2.4: Actual and Forecasting rTVIX by ARFIMA(2,d,3)-FIGARCH(1,d,1)**

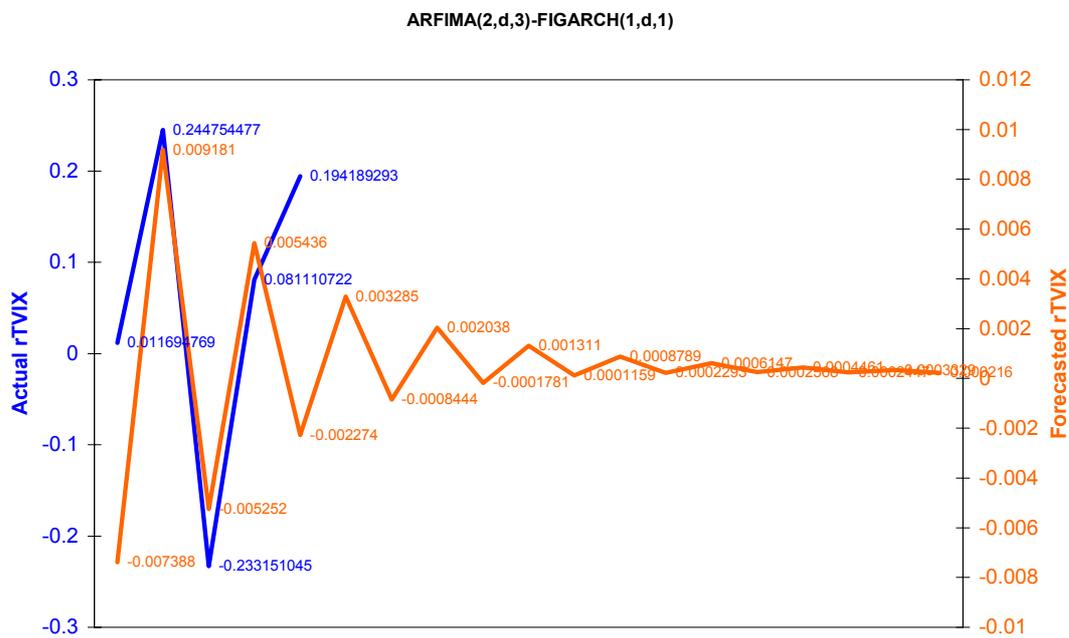


Figure 2.5: Actual and Forecasting rTVIX by ARFIMA(3,d,3)-FIGARCH(1,d,1)

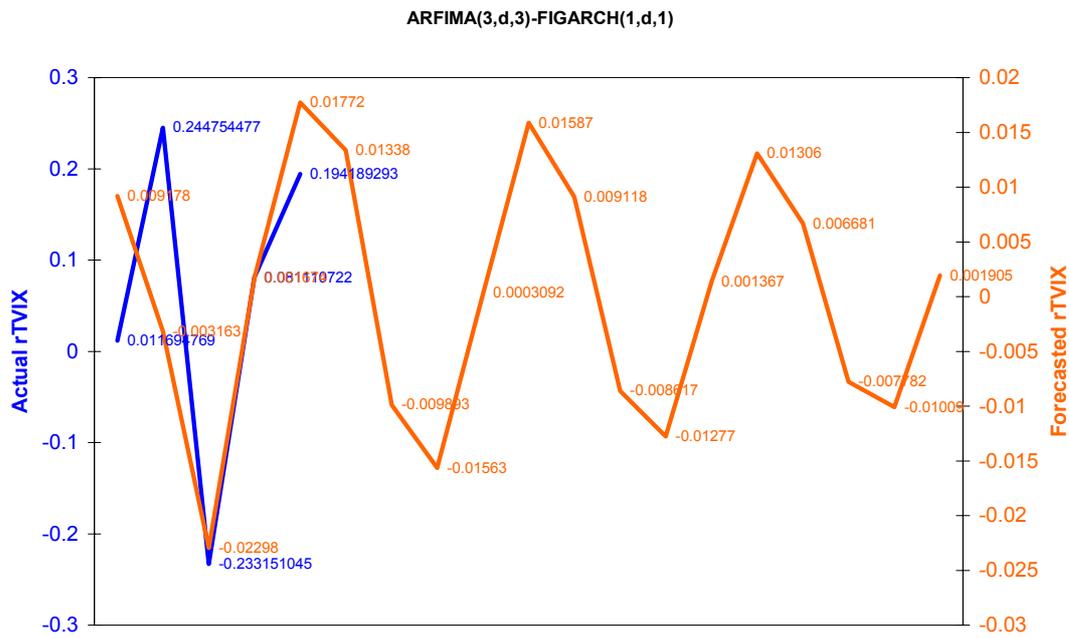
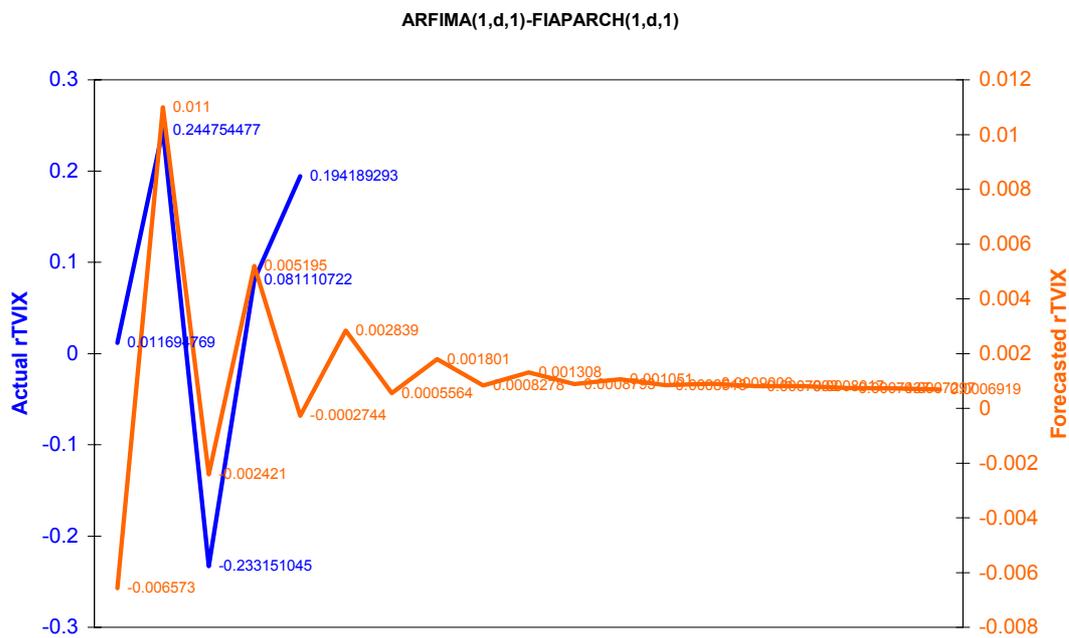
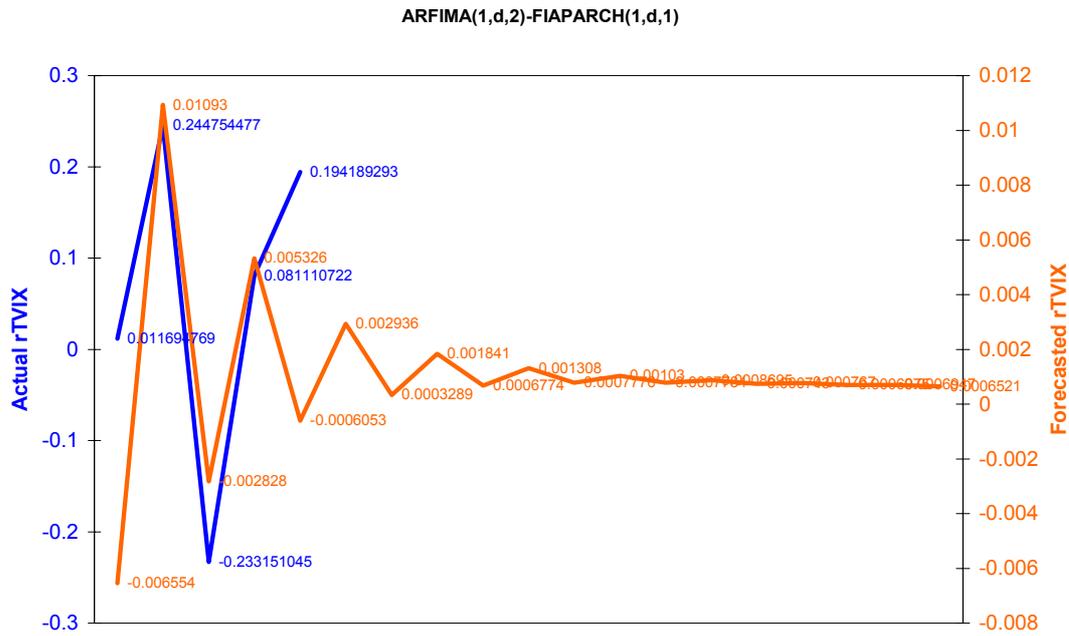


Figure 3.1: Actual and Forecasting rTVIX by ARFIMA(1,d,1)-FIAPARCH(1,d,1)



**Figure 3.2: Actual and Forecasting rTVIX by ARFIMA(1,d,2)-FIAPARCH(1,d,1)**



**Figure 3.3: Actual and Forecasting rTVIX by ARFIMA(2,d,3)-FIAPARCH(1,d,1)**

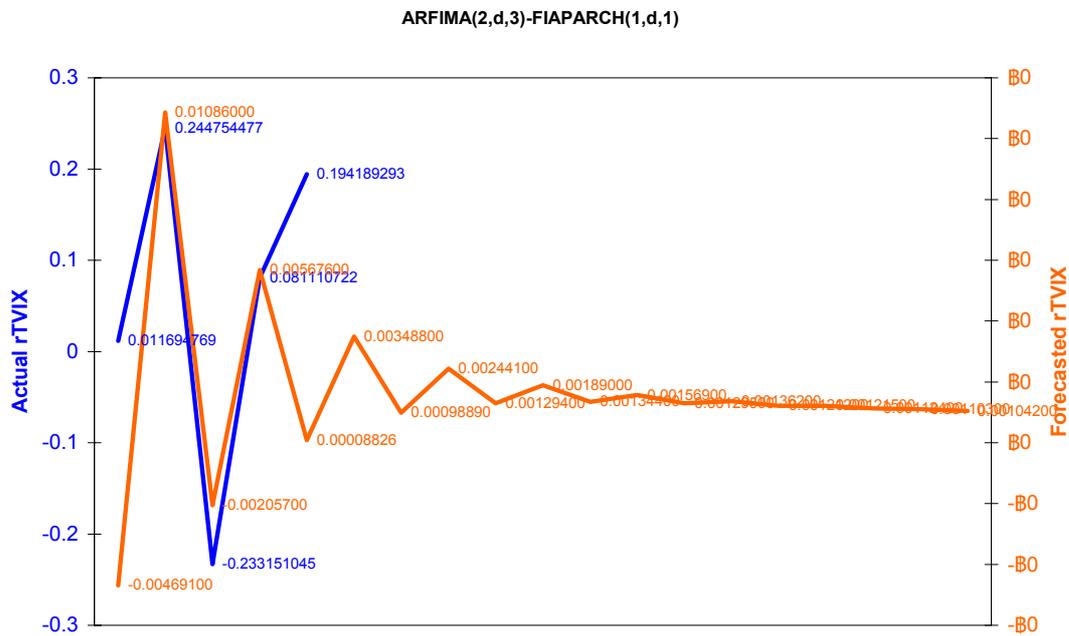


Figure 3.4: Actual and Forecasting rTVIX by ARFIMA(3,d,3)-FIAPARCH(1,d,1)

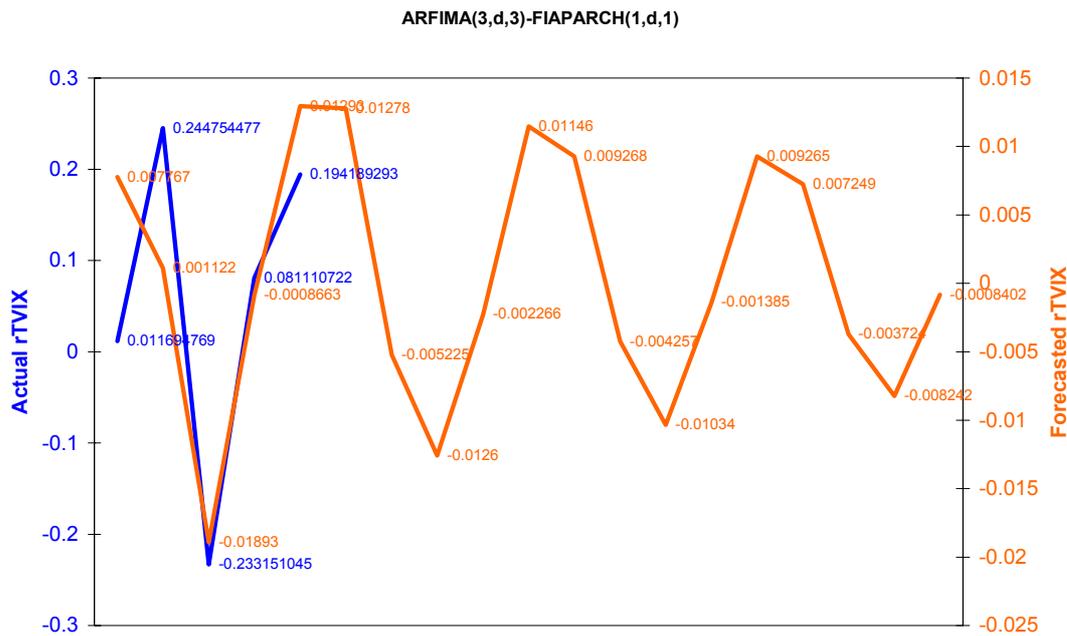


Table 5: ARMA-FIGARCH and -FIAPARCH on Returns of TVIX

	ARMA-FIGARCH		ARMA-FIAPARCH	
	Coefficient	t-value	Coefficient	t-value
AR(1)	0.7806*	6.0140 (0.000)	-0.5026	-0.9686 (0.3334)
MA(1)	-1.0190*	-6.2510 (0.000)	0.2924	0.4962 (0.6200)
MA(2)	0.1750*	2.7090 (0.0071)	-	-
$\omega$	0.0241	0.9844 (0.3256)	0.0907	0.9409 (0.3473)
A	-0.5326*	-2.8270 (0.005)	-0.4586	-0.9125 (0.3621)
B	-0.3833	-1.6130 (0.1076)	-0.3869	-0.6275 (0.5307)
$\gamma$	-	-	-0.3267	-1.1520 (0.2500)
$\delta$	-	-	1.3510*	3.498 (0.0005)
D	0.4026*	5.3820 (0.000)	0.5457*	3.7250 (0.0002)
AIC	1.886		3.856	
SBIC	29.522		35.440	

Note: \* Significant at the 1% level

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