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Comparison of Different Heavy-tailed Stochastic Volatility Models for Financial Data in Thailand

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ABSTRACT

This paper considers different stochastic volatility models to analyse the return of financial time series data in Thailand. First of all, we compare SV models with normal and Student-*t* innovations for the exchange rate of Thai Baht to the US Dollar. We show that heavy-tailed distributions are preferred in modelling the return and volatility. Secondly, we consider SV models with leverage for Thailand stock market index return and propose a modified bivariate Student-*t* distribution that allows the distribution to have marginal Student-*t* distributions with different degrees of freedom. Comparison of different SV models for the stock market index return is presented. For statistical inference, we rely on Bayesian approach using Markov Chain Monte Carlo algorithms. Student-*t* distributions are expressed into scale mixtures of normal (SMN) form to facilitate an efficient Gibbs sampling algorithm. The easy-to-use WinBUGS package is used for model implementation. We shall show that the modified Student-*t* SVL model provides the best fit to the index return data.

1. Introduction

In financial risk management, volatility of the return of a financial instrument plays an important role as a standard measure of risk. To model this volatility which is time-dependent in high frequency data, the generalised autoregressive conditional heteroskedasticity (GARCH) (Bollerslev, 1986) and stochastic volatility (SV) models are the two most popular models used. The difference between the GARCH and SV models is that the former has a deterministic volatility while the latter has a stochastic volatility. For a review of GARCH and SV models, readers are referred to Engle (1995) and Shephard (2005) for GARCH and SV models, respectively. For the SV model, Harvey (1989) proposed the following state-space form.

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Observation equation:

$$y_t \mid h_t = \exp(h_t/2)\varepsilon_t, \qquad t = 1,2,\ldots,n$$

State equation:

$$h_{t+1} \mid h_t, \mu, \phi, \sigma = \mu + \phi(h_t - \mu) + \sigma\eta, \; t = 1,2,\ldots,n-1$$

where y_t is the mean-adjusted return of an asset at time t, h_t is the unobserved log-volatility of y_t, μ measures the model instantaneous volatility which is given by $\beta = \exp(\mu/2)$, $\phi \in (-1,1)$ is the persistence in the volatility and σ is the standard deviation of the log-volatility. ε_t and η_t are standard normal processes. If ε_t and η_t are correlated with correlation coefficient ρ, the SV model has a leverage effect and we refer this model to as a SVL model. The marginal distribution of h_1 is $h_1 \mid \mu, \phi, \sigma \sim N\left(\mu, \frac{\sigma^2}{1-\phi^2}\right)$.

To implement the SV and SVL models, Bayesian approach using simulation-based Markov Chain Monte Carlo (MCMC) algorithms is recommended. For example, Jacquier *et al.* (1994) adopted the Gibbs sampling algorithm, Shephard and Pitt (1997) used the Metropolis-Hastings algorithm, Kim *et al.* (1998) reformulated the SV model to increase the efficiency of the MCMC algorithms and Meyer and Yu (2000) used Bayesian WinBUGS (Spiegelhalter *et al.*, 2004) software to implement the models.

In real applications, financial data are always heavy-tailed and the normality assumption for ε_t and η_t may be invalid. Meyer and Yu (2000) and Abanto-Valle *et al.* (2009) studied Student-*t* SV models and Chib *et al*. (2002), Omori *et al*. (2007) and Choy *et al*. (2008) studied Student-*t* SVL models. In these literatures, the Student-*t* distribution is expressed as a scale mixtures of normal (SMN) distribution (Andrews and Mallows, 1974). Choy *et al*. (2008) proposed to express the Student-*t* distribution in the SV model as a scale mixtures of uniform (SMU) distribution (Walker and Gutiérrez-Peña, 1997) to simplify the Gibbs sampler. Choy and Chan (2000) discussed the SV model with exponential power error distribution via its SMU density representation.

This paper aims to analyse exchange rate return data and stock market return data in Thailand using three Student-*t* SV-type models, namely the *t-t* SV model, *t* SVL model and modified SVL models. The *t-t* SV model allows the pair of independent errors (ε_t, η_t) to have Student-*t* distributions with different degrees of freedom (d.f.) and is particularly useful for analysing the exchange rate return data because this type of data does not exhibit a leverage effect. The *t* SVL model accounts for the leverage effect by allowing ε_t and η_t to be correlated. Under this model, the pair (ε_t, η_t) is assumed to have a bivariate Student-*t* distribution with a common d.f. and thus the marginal Student-*t* distributions have the same d.f.. Unfortunately, it is too restrictive to assume the marginal distributions are identical and in real applications, it is unlikely that the log-return and log-volatility will have the same d.f.. For this reason, this paper proposed a modified bivariate Student-*t* distribution for the SVL model which is referred to as the modified *t* SVL model. It is known that both univariate and multivariate Student-*t* distributions are SMN distributions. We suggest a minor modification to the existing SMN form of the bivariate Student-*t* distribution to create a heavy-tailed bivariate distribution which has marginal Student-*t* distributions with different d.f.. Details are given in Section 2.1.

The structure of this paper is as follows. Section 2 presents the Student-*t* distribution as a SMN distribution and proposes a modified bivariate Student-*t* distribution via the SMN representation. Three Student-*t* SV-type models are used to analyse financial data in Thailand. Section 3 presents the results of data analysis of the exchange rate return of the US dollar to Thai Baht and of the return of Bangkok Stock Exchange (BSE 30) index. For the exchange rate data, *t-t* SV model is adopted and is compared with *N-N* SV model where the distributions for ε_t and η_t are assumed to be normal. For the stock market data, we show that the modified *t* SVL model is superior to the other two models and the d.f. of the log-return and the log-volatility are very different. Finally, we conclude the paper in Section 4.

2. The Student-*t* SV and SVL Models

2.1 Student-*t* Distribution as a SMN Distribution and the Modified *t* Distribution

The class of SMN distributions is characterised by Andrews and Mallows (1974) and the Student-t is the most well known member of the SMN family. For the multivariate Student-t distribution with location vector $\boldsymbol{\mu}$, scale matrix $\boldsymbol{\Sigma}$ and d.f. ν, the joint probability density function (p.d.f) can be expressed into the following SMN representation.

$$t_\nu(\mathbf{x}\mid\boldsymbol{\mu},\boldsymbol{\Sigma}) = \int_0^\infty N(\mathbf{x}\mid\boldsymbol{\mu},\lambda\boldsymbol{\Sigma}) IG\left(\lambda \middle| \frac{\nu}{2},\frac{\nu}{2}\right) d\lambda$$

where $\mathbf{x} = (x_1,...,x_p)'$, λ is a mixing parameter of the SMN representation, $N(\mathbf{x}\mid\boldsymbol{\mu},\lambda\boldsymbol{\Sigma})$ is the multivariate normal p.d.f. and $IG\left(\lambda\middle|\frac{\nu}{2},\frac{\nu}{2}\right)$ is the inverse gamma p.d.f. In Bayesian paradigm, it is easier to rewrite the Student-t distribution hierarchically as

$$\mathbf{x}\mid\boldsymbol{\mu},\boldsymbol{\Sigma},\lambda \sim N(\mathbf{x}\mid\boldsymbol{\mu},\lambda\boldsymbol{\Sigma})$$

and $\lambda\mid\nu \sim IG\left(\lambda\middle|\frac{\nu}{2},\frac{\nu}{2}\right)$.

The univariate and bivariate Student-t distributions are special cases and the corresponding SMN forms can be obtained accordingly.

The marginal distributions of a multivariate Student-t distribution are the Student-t distributions with the same d.f.. This property may not be practical in many real applications. For this reason, we propose a modified multivariate Student-t distribution by extending the univariate mixing parameter λ to a multivariate vector $\boldsymbol{\lambda} = (\lambda_1,...,\lambda_p)'$. Define $\boldsymbol{\Lambda} = diag(\lambda_1,...,\lambda_p)$. The modified multivariate Student-t distribution has a joint p.d.f. given by

$$f(\mathbf{x}\mid\boldsymbol{\mu},\boldsymbol{\Sigma}) = \int_0^\infty \!\!...\!\int_0^\infty N\left(\mathbf{x}\mid\boldsymbol{\mu},\boldsymbol{\Lambda}^{1/2}\boldsymbol{\Sigma}\boldsymbol{\Lambda}^{1/2}\right)\prod_{m=1}^{p} IG\left(\lambda_m\middle|\frac{\nu_m}{2},\frac{\nu_m}{2}\right)d\lambda_1...d\lambda_p$$

.

It can be easily shown that the marginal distribution of x_m is a Student-t distribution with d.f. ν_m. Hence, the modified bivariate Student-t distribution can be expressed hierarchically as

$$\begin{pmatrix}x_1\\x_2\end{pmatrix} \sim N\left(\begin{pmatrix}\mu_1\\\mu_2\end{pmatrix},\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}^{1/2}\begin{pmatrix}\tau_1^2 & \rho\tau_1\tau_2\\\rho\tau_1\tau_2 & \tau_2^2\end{pmatrix}\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}^{1/2}\right)$$

and $\lambda_m \sim IG\left(\frac{\nu_m}{2},\frac{\nu_m}{2}\right), \quad m = 1,2.$

2.2 The Bayesian SV and SVL Models

In this paper, we consider the t-t SV model, t SVL model and modified t SVL model. Under these models, the bivariate distribution of $(\varepsilon_t,\eta_t)'$ are

t-t SV model: $\varepsilon_t \sim t_{\nu_1}(0,1)$ and $\eta_t \sim t_{\nu_2}(0,1)$

t SVL model:

$$\begin{pmatrix}\varepsilon_t\\\eta_t\end{pmatrix} \sim t_\nu\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & \rho\\\rho & 1\end{pmatrix}\right)$$

Modified t SVL model:

$$\begin{pmatrix}\varepsilon_t\\\eta_t\end{pmatrix} \sim \int_0^\infty\!\!\int_0^\infty N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}^{1/2}\begin{pmatrix}1 & \rho\\\rho & 1\end{pmatrix}\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}^{1/2}\right) IG\left(\lambda_1\middle|\frac{\nu_1}{2},\frac{\nu_1}{2}\right) IG\left(\lambda_2\middle|\frac{\nu_2}{2},\frac{\nu_2}{2}\right) d\lambda_1 d\lambda_2$$

In the present context, we present the models as

t-t SV model:

$$y_t \mid h_t,\lambda_{y_t} \sim N\left(0,\lambda_{y_t}e^{h_t}\right)$$

$$h_{t+1}\mid h_t,\mu,\phi,\sigma,\lambda_{h_t} \sim N\left(\mu_h,\lambda_{h_t}\sigma^2\right)$$

$$\lambda_{y_t} \sim IG\left(\frac{\nu_1}{2},\frac{\nu_1}{2}\right)$$

$$\lambda_{h_t} \sim IG\left(\frac{\nu_2}{2},\frac{\nu_2}{2}\right)$$

t SVL model:

$$\begin{pmatrix}y_t\\h_{t+1}\end{pmatrix} \sim N\left(\begin{pmatrix}0\\\mu_t\end{pmatrix},\lambda_t\begin{pmatrix}e^{h_t} & \rho\sigma e^{h_t/2}\\\rho\sigma e^{h_t/2} & \sigma^2\end{pmatrix}\right)$$

$$\lambda_t \sim IG\left(\frac{\nu}{2},\frac{\nu}{2}\right)$$

Modified t SVL model:

$$\begin{pmatrix}y_t\\h_{t+1}\end{pmatrix} \sim N\left(\begin{pmatrix}0\\\mu_t\end{pmatrix},\begin{pmatrix}\lambda_{y_t} & 0\\0 & \lambda_{h_t}\end{pmatrix}^{1/2}\begin{pmatrix}e^{h_t} & \rho\sigma e^{h_t/2}\\\rho\sigma e^{h_t/2} & \sigma^2\end{pmatrix}\begin{pmatrix}\lambda_{y_t} & 0\\0 & \lambda_{h_t}\end{pmatrix}^{1/2}\right)$$

$$\lambda_{y_t} \sim IG\left(\frac{\nu_1}{2},\frac{\nu_1}{2}\right)$$

$$\lambda_{h_t} \sim IG\left(\frac{\nu_2}{2},\frac{\nu_2}{2}\right)$$

where $\mu_h = \mu + \phi(h_t - \mu)$. To complete the Bayesian framework, we adopt the following prior distributions.

t-t SV model: $\mu \sim N(\alpha_\mu, \beta_\mu)$,
$\phi^* \sim Be(\alpha_\phi, \beta_\phi)$, $\sigma^2 \sim IG(\alpha_\sigma, \beta_\sigma)$
$\nu_1^{-1} \sim U(a_1, b_1)$, $\nu_2^{-1} \sim U(a_2, b_2)$

t SVL model: $\mu \sim N(\alpha_\mu, \beta_\mu)$,
$\phi^* \sim Be(\alpha_\phi, \beta_\phi)$, $\sigma^2 \sim IG(\alpha_\sigma, \beta_\sigma)$
$\nu^{-1} \sim U(\alpha_\nu, \beta_\nu)$, $\rho \sim U(\alpha_\rho, \beta_\rho)$

Modified SVL model: $\mu \sim N(\alpha_\mu, \beta_\mu)$,
$\phi^* \sim Be(\alpha_\phi, \beta_\phi)$, $\sigma^2 \sim IG(\alpha_\sigma, \beta_\sigma)$
$\nu_1^{-1} \sim U(a_1, b_1)$,
$\nu_2^{-1} \sim U(a_2, b_2)$, $\rho \sim U(\alpha_\rho, \beta_\rho)$.

where $\phi^* = \dfrac{\phi + 1}{2}$.

3. Data Analysis

In this section, we analyse the exchange rate data of the US dollar to Thai Baht and the Bangkok Stock Exchange (BSE 40) index data, respectively. Since many literatures reveal that exchange rate return data do not have a leverage effect and our preliminary study using a *t* SVL model for the exchange rate return of the US dollar to Thai Baht shows an insignificant leverage effect, we model the log-return data using the *t-t* SV and *N-N* SV models and compare the performance of the two models. For stock market return data, leverage is a common effect. We analyse the BSE 40 return data using two different Student-*t* SVL models and a *t-t* SV model for comparison.

In the data analysis, we assign a vague prior to μ, i.e. $\beta_\mu \to \infty$, a non-informative prior to σ^2, i.e. $\alpha_\sigma = \beta_\sigma = 0$ and $\phi^* \sim Be(20, 1.5)$ in the *t-t* SV, *t* SVL and modified *t* SVL models. For the *t* SVL and modified *t* SVL models, the prior distribution for ρ is the uniform $U(-1,1)$ distribution. The prior distribution for the d.f. is the uniform $U(0.025,1)$ distribution. That is, $\nu^{-1} \sim U(0.025,1)$ for the *t* SVL model and $\nu_1^{-1} \sim U(0.025,1)$ and $\nu_2^{-1} \sim U(0.025,1)$ for the *t-t* SV and modified *t* SVL models.

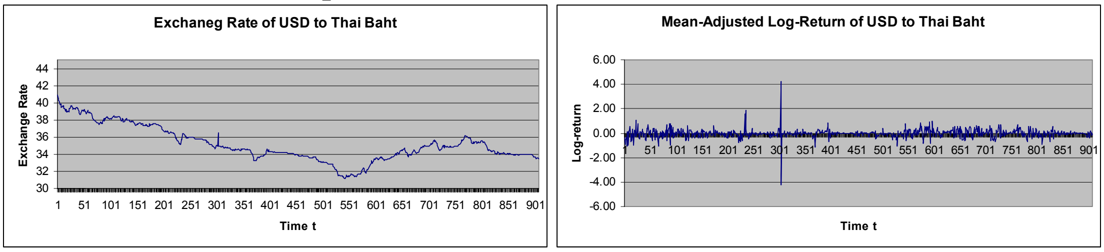
Bayesian model comparison relies on the deviance information criterion (DIC) which is designed for model comparison using MCMC outputs. See Spiegehalter *et al.* (2002) for the development of DIC. The model with a smaller DIC value is better and is preferred.

The use of SMN representation for the Student-*t* distribution can not only simplify the Gibbs sampler but also allows the outlying log-returns and log-volatilities to be identified. When expressing the Student-*t* distribution as a SMN, each y_t and each h_t will have mixing parameters, namely λ_{y_t} and λ_{h_t}, respectively, and they are used as a proxy to identify outlying returns and outlying volatilities. Large λ_{y_t} and λ_{h_t} values are associated with potential outliers. For the *t* SVL model, the pair (y_t, h_{t+1}) has a common λ_t which is used to identify outlying pairs. See Choy et al. (2008) for technical details.

In the model implementation using WinBUGS software, a Gibbs sampler was run for 10,000 iterations during the burn-in period. After the burn-in period, a further 50,000 iterations random drawings were obtained and from which a sample of size 2,000 was taken systematically at every 25th iteration to mimic a random sample from the intractable joint posterior distribution for posterior analysis.

3.1 Exchange rate of US Dollar to Thai Baht

Figure 1: The exchange rate and the log-return of USD to Thai Baht from 3 January 2006 to 30 September 2009.

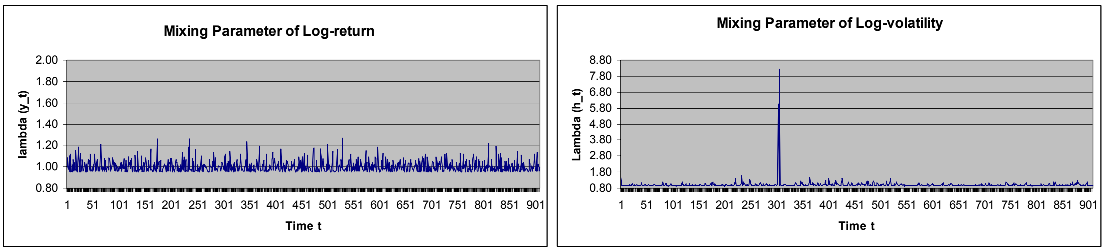


The daily exchange rate and its mean-adjusted log-return of the US dollar to Thai Baht from 3 January 2006 to 30 September 2009 are displayed in Figure 1. A preliminary study using SVL models with bivariate Student-*t* and bivariate normal distribution for the pair (ε_t, η_t) does not support a significant leverage effect and this finding is consistent with those findings from many other literatures that most exchange rate return data do not have a leverage effect (Choy *et al.*, 2008). Therefore, we analyse the data using the *t-t* SV and *N-N* SV models. Table 1 presents posterior summaries of main model parameters and the DIC values of the two models. First of all, the *t-t* SV model has a smaller DIC value and hence it is superior to the *N-N* SV model. The d.f. for the log-return and log-volatility are 23.94 and 3.09, respectively. This means that the log-volatility distribution is much heavier-tailed than the log-return distribution. In addition, the *t-t* SV model fits the data better because the Bayes estimate of the log-volatility, σ, is smaller than that of the *N-N* SV model. Both μ and ϕ are quite insensitive to the distributional assumption. Figure 2 gives the time series plots of the posterior means of the mixing parameters λ_{y_t} and λ_{h_t}. We observe no extremely large values of λ_{y_t} and there are no outlying returns. However, the graph reveals two outlying volatilities on Friday, 30/3/2007 and Monday, 2/4/2007 where the exchange rate suddenly jumped up by approximate 4.3% on the Friday and dropped by approximate 4.2% on the following Monday. It can be seen in Figure 1 that the exchange rate experienced a sudden surge on 30/3/2007 and a fall on 2/4/2007. The *t-t* SV model identifies that these two observations contribute to outlying volatilities and not to outlying returns and protects the statistical inference from the distorting effects of these outliers.

Table 1: DIC values and posterior means (standard errors in parentheses) of selected model parameters of the *N-N* SV and *t-t* SV models for exchange rate return data.

Model	DIC	μ	σ	ϕ	ν_1	ν_2
N-N SV	-170.09	-3.166 (0.283)	0.605 (0.060)	0.922 (0.019)	NA	NA
t-t SV	-175.58	-3.123 (0.328)	0.336 (0.082)	0.936 (0.019)	23.94 (8.26)	3.09 (1.27)

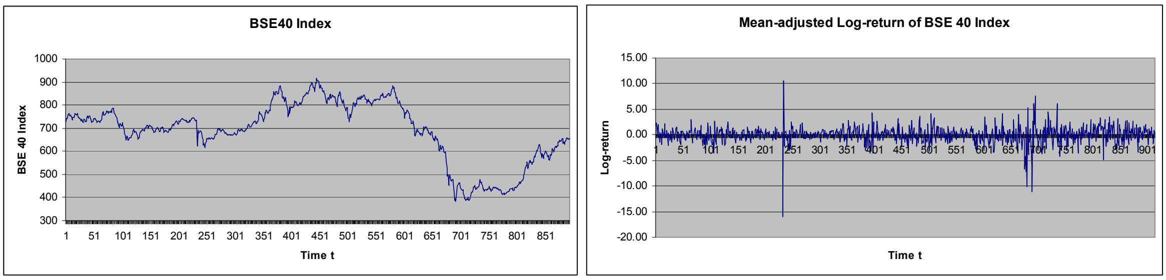
Figure 2: Posterior means of λ_{y_t} and λ_{h_t} of the t-t SV model for the exchange rate return data.



3.2 Bangkok Stock Exchange Index

In this subsection, we analyse the Bangkok Stock Exchange (BSE 40) index collected from 3 Jan 2006 to 30 September 2009. The index and the mean-adjusted log-return of the index are given in Table 2. Note that the BSE 40 index experienced a significant drop from 730.55 points to 622.14 (or 14.8%) on 19 December 2006, followed by a strong rebound to 691.55 (or 11.1%) on the following day. The significant drop was caused by the Bank of Thailand's decision to control the capital inflow on 19 December 2006. Such a policy was highly criticised by the public and was eventually redrawn in March 2008 but the initial impact was enormous. The effect on the log-return is clearly displayed in Figure 3. The corresponding data points are obvious outliers and must be handled with care in the analysis.

Figure 3: The Bangkok Stock Exchange index and the log-return of the index from 3 January 2006 to 30 September 2009.



Unlike the exchange rate data, stock market data are found to have a leverage effect in many literatures. Table 2 presents the DIC and parameter estimates of the *t-t* SV, *t* SVL and modified *t* SVL models for the BSE 40 return data. First of all, according to the DIC values, the *t-t* SV model has the largest DIC amongst the three models because of its incapability of accounting the leverage effect. The *t* SVL model has a bigger DIC value than the modified *t* SVL model and the combined d.f. is estimated to be 8.87. This model is commonly used to replace the normal SVL model for robustness consideration but the drawback is that the marginal Student-*t* distributions must have the same d.f.. From the *t-t* SV model, the estimated d.f. for the log-return and log-volatility are 23.79 and 1.73, respectively. Therefore, it is very unlikely that the marginal distributions of the log-return and log-volatility will have the same d.f. if the leverage effect is taken into account. The proposed modified *t* SVL model can account for the leverage effect and at the same time allows for different marginal Student-*t* distributions for the log-return and log-volatility. The DIC of the modified t SVL model is the smallest amongst the three models and this verifies that the modified t SVL model is superior to the other two models. The posterior means of the d.f. are 20.54 for log-return and 1.96 for log-volatility which are significantly different from the combined d.f. of 8.87 under the t SVL model but are close to those of the t-t SV model.

Table 2: DIC values and posterior means (standard errors in parentheses) of selected model parameters of the *t-t* SV, *t* SVL and modified *t* SVL models for SBE 40 return data. The last row gives the corresponding values when the data on 19/12/2006 and 20/12/2006 are removed from the analysis but its DIC value cannot be used to compare with the other three models because of different number of observations used.

Model	DIC	μ	σ	ϕ	ρ	ν_1	ν_2
t-t SV	3,069.9	0.255 (0.219)	0.0675 (0.0305)	0.971 (0.012)	NA	23.79 (8.49)	1.73 (0.49)
t SVL	3,036.3	0.282 (0.226)	0.1801 (0.0357)	0.967 (0.128)	-0.286 (0.097)	8.87 (3.44)	NA
Modified t SVL	3,006.6	0.263 (0.201)	0.093 (0.022)	0.963 (0.013)	-0.519 (0.144)	20.54 (8.64)	1.96 (0.38)
Modified t-t SVL*	3,065.0	0.398 (0.231)	0.118 (0.028)	0.971 (0.011)	-0.444 (0.128)	25.19 (7.76)	3.68 (2.28)

Figure 4: Time series plots of estimated volatilities under the (1) *t-t* SV model, (2) *t* SVL model and (3) modified *t* SVL model. The $\ln y_t^2$ is the realised volatility.

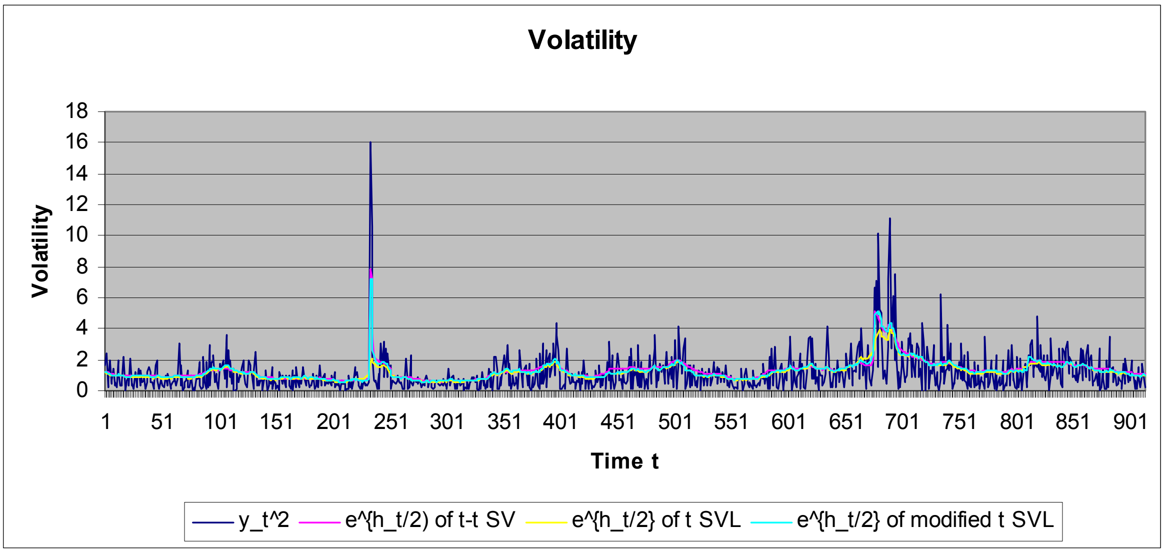
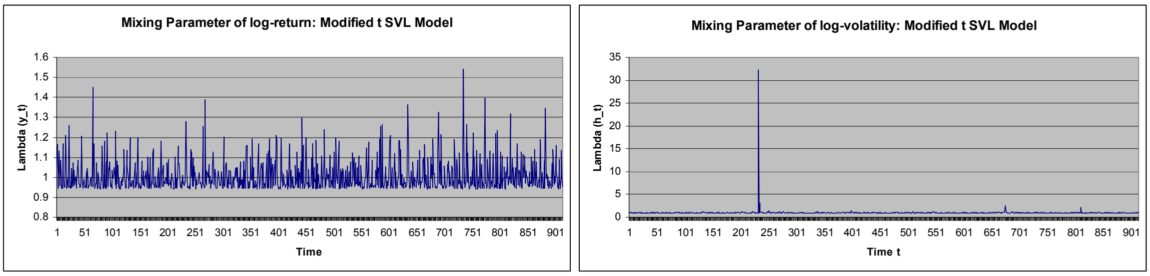


Figure 5: Posterior means of λ_{y_t} and λ_{h_t} of the modified *t* SVL model for the BSE 40 return data.



For the unobserved volatilities, they are estimated using different models and are presented in Figure 4. The realised volatility is the square of the log-return. Although Figure 4 shows that the volatilities are very close under the three models, the *t-t* SV and modified *t* SVL models provide a better fit to large realised volatilities.

It is mentioned beforehand that the data on 19/12/2006 and 20/12/2006 are questionable. The mixing parameters of the modified *t* SVL model in Figure 5 reveal that these data are outlying in log-volatility only. For comparative purpose, we delete the data on 19/12/2006 and 20/12/2006 and reanalyse the data using the modified *t* SVL model. The results in Table 2 show that there is an increase in the instantaneous volatility and the d.f. for the log-return and log-volatility.

4. Conclusion

This paper has successfully demonstrated the implementation of the Student-*t* SV-type models using WinBUGS package. The key of implementation is to express the Student-*t* distribution as a SMN distribution. For the exchange rate data which show no leverage effect, the *t-t* SV model outperforms the *N-N* SV model and also provides a means to identify outliers in the return and volatility equations. For stock market data which have a leverage effect, the modified *t* SVL model allows the log-return and log-volatility to follow different marginal Student-*t* distributions and is therefore superior to the *t* SVL model which forces the log-return and log-volatility to have the same marginal distribution.

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