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The optimal margin setting:
The application of bivariate EVT method

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ABSTRACT

Futures margin setting is important for an exchange policy since it is the balance between the loss and benefit of an exchange. This paper develops a method for setting the margin level for two-commodity portfolios. The bivariate extreme value theory is applied in our estimation to compute the margin level for a given probability of margin failure desired by brokers and exchanges. An empirical study use the return of the seven commodities (indexes) futures in the CBOT Exchange to explain our method, and then the explicit margin levels are given for every portfolio. For our specific example, the comparison of BEVT method, normal distribution and historical data are also provided.

Keywords: Future margins, exchange policy, optimal margin setting, bivariate EVT, commodities

1. Introduction

In futures market, the brokers and exchanges require customers to provide security deposits, named margins when they trade. The minimum margin levels are set by exchanges and brokers could set margin requirements for their customers beyond the minimum.(Fishe, Goldberg, Gosnell, and Sinha,1990) when the market crisis comes, the margin will make up the loss for the brokers, but if the loss is beyond the margin requirement, the difference between the price change and the margin will harm brokers, if the loss is huge and many customers involve it, it will destroy brokers and also exchanges.

Too low level margins make exchange or brokers in the risk of customers' defaults, however, too high level margin make the leverage effect disappear, and also the investors, especially arbitragers leave the market. (Booth, el., 1997) Therefore, futures margin setting is important for an exchange policy since it is a balance between losses and benefits. How to set the margin level appropriately is a big issue in an exchange's strategy (Hardouvelis and Peristiani, 1992).

2. Literature review

There are a large number of theoretical and empirical researches on the margins setting problems. Two main trends of margin setting methods are reviewed here, Fenn and Kupiec(1993) consider the margin level as a unique risk management tool, they believe the active margin setting which could affect the volatility of futures markets. They develop two models to verify that the optimal margin policy could be designed to cost-minimizing considerations, minimize contracting costs which include margin costs, settlement costs, and the cost of allowing a deficit. However, there are two problems of this kind of model: first, the cost of settlement-failure is not obvious, even the loss is greater than margin costs, the default is not necessary to happen, the margin setting is related to the individual's utility function, which is difficult to predict. Second, the empirical studies from Longin(2000), Kupiec(1997) do not give a support to the theory, the relationship between the volatility and margin level is not significant. Therefore, the design of margin setting in our paper follows the second trends, which does not consider the unknown behaviors of the investors and the volatility of the markets, but the minimum margin level, which only focus on whether the probability of a loss is large enough to deplete margin.

Figlewski (1984) suggests that the appropriate margin level should depend on many factors, one of important one is that the probability of a margin violation. Figlewski (1984), Gay et al.(1986) adopt the method which assume the price change followed the normal distribution, they derive the probability of the margin violation occurring on a given date for a given margin level. However, there always exists heavy-tailed in financial data, the normal distribution hypothesis is not that appropriate for the margin setting, since the margin violation is sensitively related to the tails. (Tsay, 2010) only the extremes offset the margin requirements and could make the loss to the exchanges or the brokers. Longin(1999) first develops a new method which is based on "Extreme Value Theory(EVT)", instead of normal distribution, the paper shows that the EVT method can grasp the extreme movements of financial data, and provide the accurate explicit optimal margin level compare with normal distribution model.

Within last decade the futures market is changing, as studies (Cater, 2001) suggest that only a small ratio of trading in futures market is hedging, the most of the long or short actions in the exchanges are speculative. Investors will not only buy one index or one commodity to safeguard the risk as a hedger, they would like to take more positions in different futures markets, long one corn, short one T-Note bond, or long(short) two biofuels,…etc. The co-movement of the futures price would lead to big losses, for instance, if one futures price in an exchange is positive correlated to another one, one futures margin failure will be likely to happen together with another. The examples of Oct. 2008, in one day on Oct. 28 the corn and wheat futures simultaneously increase more than 7%, and 8%, the customers who short these two commodities one contract each, would have to lose about \$150,000 and \$237,500 respectively. This example demonstrates that extreme price movements in one futures contract may spread quickly to other commodities.

3. Methodology

The traditional margin-setting methods ignore the co-movement of the commodities; they consider the probability of single commodity margin failure (such as S&P and Nikkei) independently. However that should not be the case, we should not set the margins separately but consider the whole account. In our common sense, if the customers have two commodities which positively correlated in one account, the margin setting should be higher than the sum of two individual margins due to the higher risk of two margins fail together; whereas they are negatively correlated, the margin for an account could be lower than the sum. "How to set margin level for an account, which takes two positions in different markets?" We solve these problems in next section.

3.1 What happen when the price is correlated?

Now we use the mathematic notations to explain our problems. Note that whether we are long or short in the futures markets, the margin level only relates to the absolute value of price change. Let the margin level as ML, the price change x, when in the short position, and when in the long position.

If the two futures prices are independent, therefore

$$P_{ind}(x_1 > ML_1 \ and \ x_2 > ML_2) = P(x_1 > ML_1) \times P(x_2 > ML_2) \qquad (1)$$

Where $P(x_i > ML_i)$ (i=1, 2) is the probability that the price change is greater than the margin level, i.e. the chances of the margin failure. And $P(x_1 > ML_1 \ and \ x_2 > ML_2)$ is the probability that both commodity price changes are greater than the margin levels, that is, the chance that margins fail together. Even there is no dependence between the two commodities, there exists possibility that both margin failures happen. For example, if we set each margin failure rate as 10%, the joint possibility will be 1%. However, if two futures are correlated positively, the problem will enlarge,

$$P_{dep}(x_1 > ML_1 \text{ and } x_2 > ML_2) = P(x_1 > ML_1 \mid x_2 > ML_2) \times P(x_2 > ML_2) \\ > P(x_1 > ML_1) \times P(x_2 > ML_2) \quad (2)$$

The problem comes from the conditional distribution is greater than the to the joint probability of margins failures, which has larger probability when have no dependence because

$$P_{dep}(x_1 > ML_1 \text{ and } x_2 > ML_2) = P(x_1 > ML_1 \mid x_2 > ML_2) \times P(x_2 > ML_2) \\ > P(x_1 > ML_1) \times P(x_2 > ML_2) \quad (3)$$

3.2 Theoretical Foundations of Margin Setting by EVT Method

We introduce the methods to design the margin level by reviewing the EVT theory, and then apply them to our examples.

3.2.1 Setting Margin Level for Individual Commodity--Univariate Extreme Value Theory (EVT)

If one commodity price change follow the normal distribution, we can calculate the individual futures i margin level (ML_i) given a certain probability of failure, such as 5%, i.e., $\Phi^{-1}(0.95)$, which is the 95% quantile of the normal distribution. However, the study shows that the distribution of price change is not always the normal distribution, extreme changes (or the heavy tails) exist in the financial series. Since the extremes price variations directly lead to margin failures, we consider the EVT because it focuses on the tail of a distribution.

Suppose there are n continuous days, for one single commodity, we would model the probability of margin failure, which is price changes during the n days will exceed the margin level:

$$\pi_i = P(M_i > ML_i) = 1 - F^n(ML_i) = 1 - G(ML_i) \quad (4)$$

Where M_i is the maxima sequence, which equal to $M_i = \max(x_{i1}, x_{i2}, ..., x_{in})$, i is the number of blocks. F is the general price distribution, and G is the asymptotic extreme value distribution. The EVT tells us that find a sequence of a_n and b_n, the maxima can be generalize to be general extreme value distribution (GEV) G: (Coles, 2001, Beirlant, 2004)

$$G(x; b, a, \xi) = \exp\left\{ -\left[1 + \xi \left(\frac{x - b}{a} \right) \right]^{-1/\xi} \right\} \quad (5)$$

Where b is the location parameter, a is the scale parameter and ξ is the shape parameter. The shape parameter ξ governs the tail behaviors of the distribution. The sub-families defined by $\xi = 0$, $\xi > 0$ and $\xi < 0$ correspond, respectively, to the Gumbel, Fréchet and Weibull families, the Fréchet family is the heavy-tail distribution.(Embrechts, et.al., 1997) The GEV distribution has a good property that it is max-stable, that is, we can derive the general distribution from the extremes distribution.

If we want to derive margin level at the 5% margin failure, let π_i equal to the 5%, margin level is equal to $G^{-1}(0.95)$, where the G^{-1} is the qunatile of GEV distribution.

3.2.2 Setting Margin Level for an Account-- Bivariate Extreme Value Theory (BEVT)

Before explaining the way to set the margin, we first introduce the Bivariate Extreme Value Theory (BEVT) briefly.

Suppose that $(X_{11}, X_{21}), (X_{12}, X_{22}), \ldots (X_{1n}, X_{2n})$ is the sequence of the two commodities price change vectors, which have distribution function $F(x_1, x_2)$. As in univariate case, the extremal behavior of multivariate extremes is based on the limiting behavior of block maxima. We define the maxima of two vectors as:

$$M_{1,n} = \max_{i=1,\ldots,n} \{X_{1i}\} \quad \text{and} \quad M_{2,n} = \max_{i=1,\ldots,n} \{X_{2i}\}$$

$$M_n = (M_{1,n}, M_{2,n})$$

M_n is the vector of component-wise maxima, it should be noted that the M_n need not be a sample point, that is, we select the maxima from the each block of single commodities, and form the component-wise maxima M_n.

The asymptotic theory of multivariate extremes goes to the analysis of M_n, as $n \to \infty$. This issue is partly resolved by recognizing that $\{X_{1i}\}$ and $\{X_{2i}\}$ considered separately are sequences of independent, univariate random variables. Consequently, standard univariate extreme value results apply to both components. For the simplicity, we transform to be the standard Frechet distribution, with distribution function:

$$F(z) = \exp(-1/z), \quad z > 0 \tag{6}$$

Which is b=0, a=1 and $\xi = 1$ in GEV distribution, since the GEV distribution has the max-stable property, we have

$$P(M_n / n) = \exp(-1/z), \quad z > 0 \tag{7}$$

We should consider the re-scaled vector

$$M_n^* = \left(\max_{i=1,\dots,n}\{X_{1i}\}/n, \max_{i=1,\dots,n}\{X_{2i}\}/n \right) \tag{8}$$

Then the Bivariate extreme value theory can prove that if the

$$P\left\{M_{1n}^* \le x_1, M_{2n}^* \le x_2\right\} \xrightarrow{d} G(x,y) \tag{9}$$

where G is a non-degenerate distribution function, G has the form

$$G(x_1,x_2) = \exp\left\{-V(x_1,x_2)\right\}, \qquad x_1 > 0, x_2 > 0 \tag{10}$$

Where $V(x_1,x_2) = 2\int_0^1 \max\left(\frac{w}{x_1}, \frac{1-w}{x_2}\right) dH(w)$ (11)

And H is a distribution function on [0,1] satisfying the mean constraint

$$\int_0^1 w dH(w) = 1/2 \tag{12}$$

The family of distributions that arise as limits in (10) is named the class of bivariate extreme value distribution.

Since the GEV family provides the complete class of marginal limit distributions, it follows that the complete class of bivariate limits is obtained simply by generalizing the marginal distribution. Specifically, letting

$$z_1 = \left[1 + \xi_1\left(\frac{x - b_1}{a_1}\right)\right]^{1/\xi_1} \text{ and, } z_2 = \left[1 + \xi_2\left(\frac{x - b_2}{a_2}\right)\right]^{1/\xi_2}$$

The complete family of bivariate extreme value distribution, with arbitrary GEV margins, has distribution function of the form

$$G(x_1, x_2) = \exp\{-V(z_1, z_2)\} \tag{13}$$

Provided $\left[1+\xi_1\left(x-b_1\right)/a_1\right]>0$, $\left[1+\xi_2\left(x-b_2\right)/a_2\right]>0$, and where the function V satisfies (11) for some choice of H. The marginal distributions are GEV with parameters (a_1,b_1,ξ_1) , (a_2,b_2,ξ_2) respectively.

It is easy to check that

$$G^n(x_1,x_2)=G(n^{-1}x_1,n^{-1}x_2)$$

for n=2, 3,..., so the BEVT distribution also possesses the property of max-stability , that is, if (X_1, X_2) has distribution G, then M_n also has distribution function G, apart from a rescaling by n^{-1}. In particular, any distribution function H on [0,1] satisfying the mean constraint (12), give rise to a valid limit , we introduce following nine BEVT parametric estimation model:

3.2.3 Nine Parametric Models

3.2.3.1 Logistic Model (Gumbel, 1960)

The bivariate logistic distribution function with dependence parameter r is

$$G(z_1,z_2)=\exp\left(-\left(x_1^{1/r}+x_2^{1/r}\right)^r\right)$$

Where $0<r\le 1$. This is a special case of the bivariate asymmetric logistic model. Complete dependence is obtained in the limit as r approaches zero. Independence is obtained when r=1.

3.2.3.2 Asymmetric Logistic Model

This distribution function with dependence parameters r and asymmetric parameters (t_1 and t_2):

$$G(z_1,z_2)=\exp\left(-(1-t_1)x_1-(1-t_2)x_2-\left[\left(t_1x_1\right)^{1/r}+\left(t_2x_2\right)^{1/r}\right]^r\right)$$

Where $0<r\le 1$ and $0\le t_1,t_2\le 1$. When $t_1=t_2=1$ the asymmetric logistic model is equivalent to the logistic model. Independence is obtained when either $r=1,\ t_1=0\ or\ t_2=0$. Complete dependence is obtained in the limit when $t_1=t_2=1$ and r approaches zero. Different limits occur when t_1 and t_2 are fixed and r approaches zero.

3.2.3.3 Husler-Reiss Model (Husler and Reiss, 1989)

The Husler-Reiss distribution function with dependence parameter r is

$$G(z_1, z_2) = \exp\left\{-x_1\phi\left[r^{-1} + r[\log(x_1/x_2)]/2\right] - x_2\phi\left[r^{-1} + r[\log(x_2/x_1)]/2\right]\right\}$$

Where $\phi(\cdot)$ is the standard normal distribution function and r>0. Independence is obtained in the limit as r approaches zero. Complete dependence is obtained as r tends to infinity.

3.2.3.4 Negative Logistic Model

The bivariate negative logistic distribution function with dependence parameter r is

$$G(z_1, z_2) = \exp\left\{-x_1 - x_2 + \left[x_1^{-r} + x_2^{-r}\right]^{(-1/r)}\right\}$$

Where r>0. This is a special case of the bivariate asymmetric negative logistic model. Independence is obtained in the limit as r approaches zero, complete dependence is obtained as r tends to infinite.

3.2.3.5 Asymmetric Negative Logistic Model (Joe, 1990)

The bivariate asymmetric negative logistic distribution function with dependence parameter r and asymmetric parameters (t_1, t_2)

$$G(z_1, z_2) = \exp\left\{-y_1 - y_2 + \left[(t_1 y_1)^{-r} + (t_2 y_2)^{-r}\right]^{(-1/r)}\right\}$$

Where r > 0 and 0 < t_1,t_2 <= 1. When $t_1 = t_2 = 1$ the asymmetric negative logistic model is equivalent to the negative logistic model. Independence is obtained in the limit as either r, t_1 or t_2 approaches zero. Complete dependence is obtained in the limit when $t_1 = t_2 = 1$ and r tends to infinity. Different limits occur when t_1 and t_2 are fixed and r tends to infinity. The earliest reference to this model appears to be Joe (1990), who introduces a multivariate extreme value distribution which reduces to $G(z_1,z_2)$ in the bivariate case.

3.2.3.6 Bilogistic Model

The bilogistic distribution function with parameters α and β is

$$G(z_1, z_2) = \exp\left\{-x_1 q^{(1-\alpha)} - x_2(1-q)^{(1-\beta)}\right\}$$

Where $q = q(x_1, x_2; \alpha, \beta)$ is the root of the equation.

$$(1+\alpha)x_1q^{\alpha} - (1+\beta)x_2(1-q)^{\beta} = 0$$

$\alpha > 0$ and $\beta > 0$. When $\alpha = \beta$ the bilogistic model is equivalent to the logistic model with dependence parameter $r = \alpha = \beta$. Complete dependence is obtained in the limit as $\alpha = \beta$ approaches zero. Independence is obtained as $\alpha = \beta$ tends to infinity, and when one of α, β is fixed and the other tends to infinity. Different limits occurs when one of α, β is fixed and the other approaches zero.

3.2.3.7 Negative Bilogistic Model (Coles and Tawn, 1994)

The negative bilogistic distribution functions with parameter α and β is

$$G(z_1, z_2) = \exp\left\{-x_1 - x_2 + x_1q^{(1+\alpha)} + x_2(1-q)^{(1+\beta)}\right\}$$

Where $q = q(x_1, x_2; \alpha, \beta)$ is the root of the equation.

$$(1+\alpha)x_1q^{\alpha} - (1+\beta)x_2(1-q)^{\beta} = 0$$

$\alpha > 0$ and $\beta > 0$. When $\alpha = \beta$ the negative bilogistic model is equivalent to the negative logistic model with dependence parameter $r = 1/\alpha = 1/\beta$. Complete dependence is obtained in the limit as $\alpha = \beta$ approaches zero. Independence is obtained as $\alpha = \beta$ tends to infinity, and when one of α, β is fixed and the other tends to infinity. Different limits occurs when one of is fixed and the other approaches zero.

3.2.3.8 Coles-Tawn Model (Coles and Tawn, 1991)

The Coles-Tawn distribution functions with parameters $\alpha > 0$ and $\beta > 0$ is

$$G(z_1, z_2) = \exp\left\{-y_1\left[1 - Be(q; \alpha+1, \beta)\right] - y_2Be(q; \alpha+1, \beta)\right\}$$

Where $q = \alpha y_2 / (\alpha y_2 + \beta y_1)$ and $Be(q; \alpha, \beta)$ is the beta distribution function evaluated at q with α and β. Complete dependence is obtained in the limit as $\alpha = \beta$ tends to infinity. Independence is obtained as $\alpha = \beta$ approaches zero, and when one of α, β is fixed and the other approaches zero. Different limits occur when one of α, β is fixed and the other tends to infinity.

3.2.3.9 Amix Model (Tawn, 1988)

The asymmetric mixed distribution functions with parameter α and β has a dependence function with the following cubic polynomial form

$$A(t) = 1 - (\alpha + \beta)t + \alpha t^2 + \beta t^3$$

Where α and $\alpha + 3\beta$ are non-negative, and where $\alpha + \beta$ and $\alpha + 2\beta$ are less than or equal to one. These constraints imply that β lies in the interval [-0.5, 0.5] and that alpha lies in the interval [0, 1.5], though α can only be greater than one if β is negative. The strength of dependence increases for increasing α (for fixed β). Complete dependence cannot be obtained. Independence is obtained when both parameters are zero.

3.3 Estimation Procedure

From the original price series (x_{11}, x_{21}),…, (x_{1n}, x_{2n}) of independent vectors we form a sequence of component-wise block maxima (m_{11}, m_{21}),…, (m_{1n}, m_{2n}). Therefore, we separately fit the maxima price sequences (m_{11}, m_{12},…,m_{1n}) and (m_{21}, m_{22},…,m_{2n}) to the GEV distribution, i.e.

$$M_i \xrightarrow{d} GEV(a_i, b_i, \xi_i) \qquad \text{i=1,2}$$

Applying maximum likelihood to the separate series, we can obtain the estiamtes, denoted $(\hat{a}_i, \hat{b}_i, \hat{\xi}_i)$, for i = 1,2. Then we transform the variable into the Frechet distribution:

$$z_i = \left[1 + \hat{\xi}_i \left(\frac{x - \hat{b}_i}{\hat{a}_i}\right)\right]^{1/\hat{\xi}_i}$$

Then we generate a new sequence of independent realizations of a vector having bivariate extreme value distribution.

The probability density function of this model is

$$g(x, y) = \{V_x(x, y)V_y(x, y) - V_{xy}(x, y)\}\exp\{-V(x, y)\}, \quad x>0,\ y>0$$

Where V_x, V_x and $V_{x,y}$ denote partial and mixed derivatives of V respectively, leads to the likelihood

$$L(\theta) = \prod_{i=1}^{n} g(z_{1i}, z_{2i})$$

And the corresponding log-likelihood

$$l(\theta) = \prod_{i=1}^{n} \log g(z_{1i}, z_{2i})$$

We maximize the function based on quasi-Newton methods in our examples.

4. Data

Daily data from 2005/04/01 to 2012/04/30 is traded in CBOT exchange has been examined to justify the right level of margins, the seven commodities are wheat, soybean, biofuel, Dow Jones index, ten year T-note and also the five year T-note. The reason why we choose these seven kinds of commodities is that they have the largest trading volume in their sub-group. To get the stationary sequence, we logarithm the time t price (log (P_t)), and then take the difference to represent the price change, such that we obtain daily price change r_t = log (P_t)-log (P_{t-1}). For conducting the EVT method of last section proposed, we obtain the maxima and minima return series, hence we block the return calendar monthly to get the maxima return, and for the minima return we negative return ($-r_t$).

5. Results

5.1 Correlation of Commodities

Before finding the explicit margin level, we check the dependence of the commodities, here we do not use the whole data co-movement but the extremes, since the margin failure only related to the extremes, the question we ask is that "does there exist the co-movement of large price changes (extremes)?" since we have three combinations of long or short two commodities, we show the correlations of the extremes separately as followed.

TABLE 1. Correlation between the Maxima of Two Series

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-Note	5 Year T-Note
Wheat	X	0.369 ***	0.541 ***	0.138	0.315 ***	0.331 ***	0.287 ***
Soybean	0.253 ***	X	0.441 ***	0.142 **	0.088	0.183 *	0.165
Corn	0.375 ***	0.308 ***	X	0.254 ***	0.120 *	0.199 **	0.164
Biofuel	0.106	0.197 **	0.365 ***	X	-0.013	0.012	0.065

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-Note	5 Year T-Note
Dow Jows	0.218 ***	0.131	0.183 *	-0.012	X	0.977 ***	0.884 ***
10 Year T-Note	0.232 ***	0.120 *	0.144 **	0.007	0.888 ***	X	0.889 ***
5 Year T-Note	0.199 ***	0.118 *	0.123*	0.046	0.704 ***	0.717 ***	X

Note: * represents 10% significant level of two-sided t test; ** represents 5% significant level of two-sided t test; *** represents 1% significant level of two-sided t test

TABLE.2 Correlation between the Minima of Two Series

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-Note	5 Year T-Note
Wheat	X	0.353 ***	0.478 ***	0.187 *	0.391 ***	0.409 ***	0.379 ***
Soybean	0.238 ***	X	0.456 ***	0.071	0.225 ***	0.323 ***	0.258 ***
Corn	0.344 ***	0.303 ***	X	0.189 ***	0.03	0.067	0.198 **
Biofuel	0.127 **	0.11	0.272 ***	X	-0.083	-0.051	-0.058
Dow Jows	0.260 ***	0.322 ***	0.054	-0.053	X	0.9507 ***	0.783 ***
10 Year T-Note	0.275 ***	0.216 ***	0.047	-0.027	0.850 ***	X	0.750 ***
5 Year T-Note	0.251 ***	0.172 ***	0.142 **	-0.03	0.648 ***	0.604 ***	X

Note: * represents 10% significant level of two-sided t test; ** represents 5% significant level of two-sided t test; *** represents 1% significant level of two-sided t test

Tab.1 gives the correlation of the maxima of two commodities, Tab.2 shows the minima of two series, and Tab.3 provides the correlations of the maxima and minima of two commodities. Each cell has two parts, in order to compare the results of different dependence estimators, in the upper triangle we report the spearman dependence, and for the lower triangle the results are Kendall dependence; And the second part "*" give the results of t test's significance, which shows whether the correlation is significant. The positive (negative) value of the Kendall (Spearman)'s dependence implies the positive (negative) correlation between the extremes; the two estimators are almost consistent in our case. The positive correlation implies that if

TABLE 3. Correlation between the Maxima and Minima of Two Series

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-Note	5 Year T-Note
Wheat	X	-0.189 *	-0.167	0.076	-0.411 ***	-0.443 ***	-0.336 ***
Soybean	-0.003	X	0.054	-0.045	-0.219 ***	-0.292 ***	-0.276 ***

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-Note	5 Year T-Note
Corn	-0.166 ***	-0.07	X	-0.001	-0.142 **	-0.197 *	-0.146
Biofuel	0.011	-0.180*	-0.165	X	-0.017	-0.019	0.026
Dow Jows	-0.16 **	-0.412 ***	-0.210 **	-0.038	X	-0.556 ***	-0.548 ***
10 Year T-Note	-0.184 ***	-0.284 ***	-0.177 **	-0.035	-0.376 ***	X	-0.555 ***
5 Year T-Note	-0.190 ***	-0.228 ***	-0.180 ***	-0.034	-0.323 ***	-0.345 ***	X

Note: * represents 10% significant level of two-sided t test; ** represents 5% significant level of two-sided t test; *** represents 1% significant level of two-sided t test

Investors trade two commodities at same time, during a certain time interval (one month for our example), one margin fails, and the more possibility of the failure also happens for the other margin. Although it may not happen in one day, it shows the trend of co-movement of the extremes. It is obvious that the commodities in the same group are largely correlated, such as the wheat, soybean and corn, which are agricultures futures, and also for the ten year T-note and five year T-note they are correlated with each other, whereas different commodities in different group are not correlated or not significant. It should be noted that the biofuel futures are largely dependent on the agricultural futures because the widely application of agricultural commodities in biofuel productions in recent year.

5.2 Margin Setting Procedure

5.2.1 Example: "Short-two Account": Soybean and Corn

Now we would answer questions as "What is the fair margin level associated with a given value of the probability of margin failure for an account which has two commodities over n trading day?"

$$\pi_{dep} = P(x_1 > mar_1, x_2 > mar_2) > \pi_{ind} = P(x_1 > mar_1)P(x_2 > mar_2) \quad (14)$$

For example, if we set the margin level ML_1 for first commodities as the 90% quantile of the distribution, that is, according to our assumption that the margin failure is 10%. And then we would set the other margin for an account, like the margin 1, we set the second margin ML_2 as 90% quantile, assume there is no dependence between two commodities, equal to 0.01.

Our motivation in this paper is to find an appropriate ML_2 such that the probability of both margin failures as low as possible. For our specific problem, we want $\pi_{dep} = \pi_{ind} = 0.01$.

We know that

$$\pi = P(x_1 > mar_1, x_2 > mar_2) \\ = 1 - P(x_1 < mar_1) - P(x_2 < mar_2) + P(x_1 < mar_1, x_2 < mar_2) \tag{15}$$

Since ML_1 is known, that is,

$$P(x_2 < mar_2) - P(x_1 < mar_1, x_2 < mar_2) = 0.09 \tag{16}$$

We would solve it by the BEVT method we introduced in the last section. Since the problem is practical we will estimate the GEV estimators and bivariate EVT model parameters for every pair. Now we find a maxima dependent pair in the corn and soybean series, the correlation coefficient of Kendall is 0.308 and the Spearman is 0.441, which show that they are largely correlated. By using the EVT, we fit the maxima data into GEV distribution. In this example, we want to set 90% quantile as the margin for the first commodities, and 1% as the both margin failures.

Step1: Set Margin Requirement of Corn Futures

First, we fit the corn return data into the GEV distribution, and estimate the parameters by maximum likelihood method.

$$F(x;\hat{\mu},\hat{\sigma},\hat{\xi}) = \exp\left\{-\left[1+\hat{\xi}\left(\frac{x-\hat{b}}{\hat{a}}\right)\right]^{-1/\hat{\xi}}\right\}$$

$\hat{b}$ = 1.475, $\hat{a}$ = 0.624, $\hat{\xi}$ = -0.012;

The shape parameter is negative, which shows that the distribution is heavy tail. Find the 90% quantile of the parent distribution, which is equal to 0.959, therefore we set 0.959 as the corn margin level (first margin).

Step 2: Set Account Margin Level for Two Commodities

First, to apply the BEVT we transform the marginal data to the Frechet distribution (as the section 2), then we fit the standard data into the nine bivariate EVT models as above and estimate the joint distribution parameters by the Maximum likelihood functions, use the AIC principle to select the best model.

TABLE 4. Summary of Models

Model	Parameters	AIC
Logistic	r= 0.740	337.391
Asymm. Logis.	t_1 = 0.286; t_2 = 0.301; r=0.101	330.299
Husler-Reiss	r= 0.967	338.756
Negati. Logis.	R= 0.607	338.149
Asyms. Negati. Logis.	t_1 = 0.715; t_2 = 0.999 ; r=0.724	341.183
Bilogistic	α = 0.723; β = 0.752	339.362
Negat. Bilog.	α =1.324; β = 2.104	340.006
Coles-Tawn	α =0.425; β = 0.376	339.679
Amix	α =0.938; β = -0.215	339.029

Note that the AIC, the goodness-of-fit measure is calculated by the following equation: $AIC = -2L + 2n_p / n$. Where L is the value of the likelihood function and n_p is the number of parameters.

Therefore, we select the asymmetric negative logistic distribution function since it obtains the smallest AIC value.

$$G(z_1, z_2; \hat{t}_1, \hat{t}_2, \hat{r}) = \exp\left(-(1-\hat{t}_1)y_1 - (1-\hat{t}_2)y_2 - \left[\left(\hat{t}_1 y_1\right)^{1/\hat{r}} + \left(\hat{t}_2 y_2\right)^{1/\hat{r}} \right]^{\hat{r}} \right)$$

That is, $\hat{t}_1$=0.286, =0.301; =0.101

Second, to set the second margin to make the co-failure value equal to 0.01, which have to satisfy the equation (16). Therefore, we solve the extra margin requirement for the account is 1.941

Step 3. Compare Different Methods

To compare the different method, we sum up the margins, they are 2.900, if we reverse the sequence of the first and the second margins that is 3.261 which is higher than the first combination. According to the same mechanism, we could use the bivariate normal distribution to solve this problem. For the margin 1, suppose the data follow the normal distribution

$$F(x; \hat{\mu}_1, \hat{\sigma}) = \int_{-\infty}^{x_1} \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\hat{\mu}}{\hat{\sigma}})^2} dx \qquad (17)$$

Where $\hat{\mu}$ is mean and $\hat{\sigma}$ is the standard deviation.

We try to compute the $P(x > ML_1) = 0.1$, $ML_1 = \Phi^{-1}(0.9)$, ML_1 is equal to 1.232. And the joint distribution of the two margins which is the bivariate normal distribution:

$$F(x_1, x_2; \mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$$
$$= \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}\right]\right\} dx_1 dx_2 \tag{18}$$

We take this into the equation (16), compute the other margin, equal to 1.762. Therefore the account margin is 2.994, which is a little bit higher than the EVT method. To compare different methods, we also provide the historical method, the steps are as below, for the first margin setting:

$$\pi_{his} = \frac{Number\{x_1 > ML_1\}}{N} \tag{19}$$

In our case, $\pi_{his} = 0.1$, which is the 90% quantile, we find ML_1=1.163, and same for the co-movement failure value:

$$\bar{\pi}_{his} = \frac{Number\{x_1 > ML_1,\ x_2 > ML_2\}}{N} \tag{20}$$

Which are equal to 0.01, ML_2=1.715, therefore for this account we charge the margin level at 2.878 by historical method.

In the Table 5 we compare these three methods with different levels of margin failure. First we verify the results of EVT method with the normal distribution method, the results in “short-two-account” example are almost same, however the EVT distribution provide more accurate margin level than the normal distribution. Meanwhile, unlike the traditional method of margin setting, the second margin is higher, for instance, in EVT method to set soybean margin (second margin), we will set them as 0.816(10% failure rate), 1.182(5% failure rate), 1.968(1% failure rate) without considering the dependence of two series, they are almost half of our estimate results (1.941, 2.567, 3.921). When the extremes in financial market come, it could be a huge loss for the exchanges and brokers.

5.3 Results

In this section, we calculate the second margin levels by the method we develop above, in the Tab.6 we show the second margin levels in “Short-two Account”, Tab.7

TABLE. 5 Comparisons of Different Margin Levels Set by Different Methods

	EVT Distribution			Normal Distribution			Historical Data		
Margin Failure	1st Margin	2nd Margin	Account Margin	1st Margin	2nd Margin	Account Margin	1st Margin	2nd Margin	Account Margin
0.1	0.959	1.941	2.9	1.232	1.762	2.994	1.163	1.715	2.878
0.05	1.425	2.567	3.992	1.575	2.167	3.742	1.618	2.553	4.171
0.01	2.438	3.921	6.359	2.218	2.934	5.152	N.A	N.A	N.A

Note: the historical data is too much small in the 99% level due to the amount of total data is not enough.

provides the second margin levels in “Long-two Account” and Tab.8 contains the second margin in “Long-one-short-one Account”. It is important to note that in Short-corn-soybean account, if we change the soybean as the first margin, it will give different account margin, therefore for the fairness the exchange could average the two account margin, and make account margin balance.

There are several important findings, first, it can be shown that the stronger dependence between the two commodities (see Tab.1), the greater the second margin charge for an account. This is consistent with our common knowledge, the more dependent commodities, and the greater chances in the margin failure. Between the agricultural commodities futures, that are the corn, soybean and wheat futures, we set much higher margin than the agricultural commodities futures with other stocks index futures, such as corn and Dow Jows index.

Second, the second margins set in the “Long-Two Account” are usually higher than those in the “short-two account”, which implies that the more correlated relationship between the minima than the maxima.

Third, we notice that for the “long-one-short-one”, almost all the second margin are set below the zero since the negatively correlated between two commodities, if the customers hedge in this way, generally speaking, the loss will offset by the gain, the margin failures will not easily happen, theoretically we could charge the zero margin. Fourth, since the significant correlations exists in between stock index futures and also between the agricultural commodities(see Tab.1 and Tab.2), customers trading these futures are subject to greater margin errors, in our developed method, the margins are almost set by twice the traditional setting way.

Finally, for the customers only trade the financial futures or agriculture commodities seems too risky since there exists the joint extremes, therefore for exchanges and brokers, setting the right level of margins for these kinds of portfolio are important.

TABLE 6. Explicit Second Margin Level Setting of "Short-Two Account" by EVT Method

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-note	5 Year T-note
Wheat	X	3.152[3]	4.113[1]	2.677[1]	0.526 [7]	2.806[6]	2.702[9]
Soybean	2.456[3]	X	2.305[2]	2.426[2]	1.820[6]	1.945[6]	1.570[2]
Corn	3.355[1]	3.513[2]	X	3.070[9]	2.704[3]	2.978[8]	2.600[2]
Biofuel	1.982[9]	3.560[2]	2.485[9]	X	1.138[3]	1.138[3]	1.137[2]
Dow Jows	0.526[7]	0.540[6]	0.491[3]	0.225[3]	X	1.000[1]	0.260[2]
10 Year T-note	0.682[6]	0.457[6]	0.647[8]	0.246[3]	1.000[1]	X	0.247[2]
5 Year T-note	0.443[9]	0.205[1]	0.366[2]	0.144[3]	0.330[2]	0.297[2]	X

Note: we consider the commodities in first horizontal line as the first margin, the number in each cell is the corresponding second margin, and the numberi in the bracket is the 1-9 parametric models.[1]: Logistic Model, [2] Asymmetric Logistic Model, [3] Husler-Reiss Model, [4] Negative logistic Model, [5]) Asymmetric Negative logistic Model, [6] Bilogistic Model,[7] Negative Bilogistic Model [8] Coles-Tawn Model, [9] Amix Model

TABLE 7. Explicit Second Margin Level Setting of "Long-Two Account" by EVT Method

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-note	5 Year T-note
Wheat	X	3.200[2]	3.622[9]	2.641[3]	3.001[2]	2.787[1]	3.272[2]
Soybean	2.510[1]	X	2.871[3]	2.426[2]	1.756[3]	1.971[3]	1.570[1]
Corn	4.677[6]	3.474[3]	X	3.678[7]	2.704[3]	2.743[3]	3.197[2]
Biofuel	2.070[3]	3.56[2]	3.494[7]	X	1.587[6]	1.07[6]	1.137[3]
Dow Jows	0.579[2]	0.375[3]	0.491[3]	0.286[6]	X	0.000[2]	0.630[6]
10 Year T-note	0.616[1]	0.487[3]	0.573[3]	0.288[6]	0.000[2]	X	0.777[6]
5 Year T-note	0.57[2]	0.205[1]	0.366[2]	0.144[3]	2.000[6]	0.800[6]	X

Note: we consider the commodities in first horizontal line as the first margin, the number in each cell is the corresponding second margin, and the numberi in the bracket is the 1-9 parametric models.[1]: Logistic Model, [2] Asymmetric Logistic Model, [3] Husler-Reiss Model, [4] Negative logistic Model, [5]) Asymmetric Negative logistic Model, [6] Bilogistic Model,[7] Negative Bilogistic Model [8] Coles-Tawn Model, [9] Amix Model

TABLE 8. Second Margin Level Setting of "Short-One-Long-One-Account" by EVT

	Wheat	Soybean	Corn	Biofuel	Dow Jows	10 Year T-note	5 Year T-note
Wheat	X	-2.107[4]	-1.399[1]	-1.268[6]	-0.224[1]	-2.13[4]	-2.041[4]
Soybean	-0.633[1]	X	-0.641[5]	-0.633[1]	-0.224[1]	-0.633[1]	-0.641[5]
Corn	-1.935[7]	-1.182[3]	X	-1.136[3]	-0.224[3]	-1.936[4]	-1.919[3]
Biofuel	-1.400[3]	-2.146[7]	-2.373[3]	X	-2.159[4]	-2.145[3]	-2.164[3]
Dow Jows	-0.109[4]	-0.110[1]	-0.110[1]	-0.110[1]	X	-0.110[1]	-0.110[2]
10 Year T-note	-0.148[1]	-0.148[1]	-0.148[1]	-0.148[1]	-0.111[5]	X	-0.15[5]
5 Year T-note	-0.086[1]	-0.086[1]	-0.086[1]	-0.086[1]	-0.086[1]	-0.086[1]	X

Note: we consider the commodities in first horizontal line as the first margin, the number in each cell is the corresponding second margin, and the numberi in the bracket is the 1-9 parametric models.[1]: Logistic Model, [2] Asymmetric Logistic Model, [3] Husler-Reiss Model, [4] Negative logistic Model, [5]) Asymmetric Negative logistic Model, [6] Bilogistic Model,[7] Negative Bilogistic Model [8] Coles-Tawn Model, [9] Amix Model

6. Concluding remarks

1. There exists the statistically dependence between extreme prices changes in different futures traded in CBOT exchanges,

2. To solve the margin mis-setting problem when the customers who trade more commodities, we try to control the risk of co-movement of the two variables. This paper then gives the explicit margin level between seven commodities which traded most in CBOT exchanges by bivariate extreme value method.

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