*Thailand Econometrics Society, Vol. 3, No. 3 (January 2011), 6 - 19*

# Modeling Dependence of Seemingly Unrelated Tobit Model through Copula: A Bayesian Analysis*

Nuttanan Wichitaksorn and S.T. Boris Choy [a]

[a] Discipline of Operations Management and Econometrics, Faculty of Economics and Business, University of Sydney, Australia

ARTICLE INFO

ABSTRACT

We extend the Bayesian analysis of Seemingly Unrelated Tobit models by modeling dependence through a bivariate Gaussian copula. Having employed the copula approach, we avail ourselves of more flexibility in coupling together various distributions for marginal errors of the model. Bayesian Markov Chain Monte Carlo methods through Metropolis-Hastings coincident with data augmentation are performed to obtain the parameter estimates through a simulation study. We also apply our model and methods to Thai wage earnings data in 2002, which yield satisfactory results.

## 1. Introduction

The seemingly unrelated regressions (SUR) introduced by Zellner (1962) is a way to model dependence through multivariate regressions (Greene 2003, p.341). The SUR Tobit model, as an extension of the SUR model, is a special case where all dependent variables are partially observed, i.e., above zero. Many authors have been attempting to implement the SUR Tobit model through various estimation techniques. All previous work can be divided into two major groups according to the type of multivariate distribution. One is the multivariate normal distribution and the other is copulas.

In the first group, the estimation techniques are mainly the classical approach. Among others, Wales and Woodland (1983), Brown and Lankford (1992), and Kamakura and Wedel (2001) estimated the model by applying maximum likelihood techniques. Huang et al. (1987) used the expectation-maximization (EM) algorithm, which was subsequently extended to expectation-conditional maximization (ECM) by Meng and Rubin (1996). Huang (1999) applied the Monte Carlo method to the ECM algorithm. Trivedi and Zimmer (2005) indicated that all of these techniques are difficult to implement, especially in high-dimensional cases, and suggested this as a reason why applications of the SUR Tobit model are still limited. Huang (2001) and Taylor and Phaneuf (2009), among others, implemented the model through Bayesian approach using Gibbs samplers.

In the second group, which models the dependence of censored regressions through copulas, estimation techniques are limited to maximum likelihood alone. Trivedi and

*Prepared for the Fourth Conference of Thailand Econometric Society, January 13-14, 2011, Chiang Mai, Thailand. We would like to thank Professor Ammar Siamwalla for his useful advice and comments on the model specification and application to Thai data.

Zimmer (2005) applied various copulas to the SUR Tobit model and estimated the model parameters by using a two-step maximum likelihood procedure. We have found no authors who have implemented the copula-based SUR Tobit model by using a Bayesian approach.

In this paper, we model the dependence of an SUR Tobit model through a bivariate Gaussian copula and propose a Bayesian Markov Chain Monte Carlo (MCMC) method to estimate the model parameters. We implement our model and method through both a simulation and an empirical study using Thai wage earnings data in 2002. Both simulation and empirical studies yield satisfactory results. The organization of the paper is as follows. Section 1 provides the introduction. Section 2 presents a copula-based SUR Tobit model. Section 3 sets out the estimation method. Section 4 performs the simulation study. Section 5 presents the data and application. Section 6 concludes.

## 2. Model

### 2.1 Copulas

Sklar's theorem shows how a copula can be constructed from a multivariate distribution and its margins. If $F$ is a d-variate continuous distribution function with margins $F_1, \ldots, F_d$, then a multivariate function $C : [0, 1]^d \rightarrow [0, 1]$ is a copula for all $x_1, \ldots x_d \in R$ with

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)) = C(u_1, \ldots, u_d)$$

where $u_1, \ldots, u_d$ are the realization of $U_1, \ldots, U_d$ or $U_j \sim Unif(0, 1)$ for $j = 1, \ldots,$ d (for more details, please see Sklar(1959), McNeil et al. (2005), Nelsen (2006), and Patton (2009)). Sklar's theorem also allows the flexibility on the marginal distributions of $x_1, \ldots x_d$, which can be any continuous distribution functions $H(\cdot)$.

In this paper, we apply a bivariate Gaussian normal) copula, which can be expressed as:

$$C(u_1, u_2 \mid R) = \mathbf{\Phi}_2[\Phi^{-1}(u_1), \Phi^{-1}(u_2) \mid R]$$

where $R$ is a correlation matrix, $\mathbf{\Phi}_2$ is a bivariate standard normal cumulative distribution function (*cdf*) with a correlation matrix $R$ and $\Phi$ is the standard normal cdf. It follows from Song (2000) and Song et al. (2009) that the density of the bivariate Gaussian copula is given by:

$$c(u_1, u_2 \mid R) = \mid R \mid^{-\frac{1}{2}} \exp\{\tfrac{1}{2} q'(I_2 - R^{-1})q\},$$

where $\boldsymbol{q} = (q_1, q_2)'$ is a vector of normal scores $q_j = \Phi^{-1}(u_j)$ for $j = 1,2$ and $I_2$ is the identity matrix. Pitt *et al.* (2006) show that the data ($y_j$) can be generated through $y_j = H_j^{-1}(u_j)$ or $y_j = H_j^{-1}[\Phi(q_j)]$ so that the Gaussian copula linear regression model is given by:

$$y_{ij} = x'_{ij}\beta_j + \sigma_j q_{ij}, \text{ for i = 1, …, N,}$$

where $x_{ij}$ is a $k \times 1$ vector of covariates for marginal regression $j$, $\beta_j$ is a $k \times 1$ vector of regression coefficients, and $\sigma_j$ is the scale parameter for the standard normal error $q_j$. Let $\theta_j$ denote $(\beta_j, \sigma_j)$. Then, the likelihood of the model is given by:

$$\ell(Y \mid \theta, R) = \prod_{i=1}^{N} \mid R \mid^{-\frac{1}{2}} \exp\{\tfrac{1}{2} q'_i(I_2 - R^{-1})q_i\} \prod_{j=1}^{2} h_j(y_{ij} \mid x_{ij}; \theta_j),$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, $\mathbf{q_i} = (q_{i1}, q_{i2})'$ and $h_j(.)$ is the probability density function of marginal model $j$ (or $h_j(z) = \frac{\partial H_j(z)}{\partial z}$ )

### 2.2 SUR Tobit Model

Our bivariate SUR Tobit model is given by:

$$y_{i1}^* = x'_{i1}\beta_1 + \sigma_1 e_{i1},$$
$$y_{i2}^* = x'_{i2}\beta_2 + \sigma_2 e_{i2},$$
$$y_{i1} = y_{i1}^* \; if \; y_{i1}^* > 0$$
$$y_{i1} = 0 \; otherwise,$$
$$y_{i2} = y_{i2}^* \; if \; y_{i2}^* > 0$$
$$y_{i2} = 0 \; otherwise,$$

where $y_{i1}$ and $y_{i2}$ are observed dependent variables. In this study, we assume that $e_{i1} \sim N(0,1)$ and

$e_{i2} \sim N(0,1)$ where

$\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} \sim GC\begin{pmatrix} \Phi(e_{1i}) \\ \Phi(e_{2i}) \end{pmatrix}, R \Bigg), R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, which can be decomposed from $\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_1^2 \end{bmatrix}$ through $R = S^{-1/2}\Sigma S^{-1/2}$, where $GC$ denotes Gaussian copula, R is the correlation matrix, $\Sigma$ is the variance-covariance matrix and $S = diag(\Sigma)$.

The complication arises as the Tobit model is a mixture of continuous and discrete distributions. A copula can exist and provide uniqueness when margins are continuous but it is not unique in the case of discrete margins, as $q_{ij}$ can belong to several copulas (McNeil *et al*. 2005, p.187, Pitt *et al*. 2006). To overcome this problem, we follow Chib (1992) in using the data augmentation technique to make each Tobit model have a continuous margin. Then, we employ MCMC methods to estimate the model parameters. Our model and methods in this paper are similar to those of Huang (2001) except that Huang modeled the dependence structure through a multivariate normal distribution (not copula).

## 3. MCMC Methods

In order to make the censored distribution of the Tobit model continuous, we employ the method by Chib (1992) for data augmentation, which was initiated by Tanner and Wong (1987), together with the converse of Sklar's theorem that allows us to generate random numbers from copula (McNeil et al. (2005, p.193)).

Given the correlation matrix $R$, we first generate a vector of standard normal random numbers $\mathbf{Z} \sim \mathbf{\Phi}_2(0, R)$ where $\mathbf{Z} = (Z_1, Z_2)'$. Then, obtain a vector of standard uniform random numbers $\mathbf{U} = (\Phi(Z_1), \Phi(Z_1))'$where $\mathbf{U} = (U_1, U_2)'$. After that we use $U_1$ and $U_2$ to generate the unobserved dependent variables $y_1^* \le 0$ and $y_2^* \le 0$ through $y_{ij}^* = x_{ij}'\beta_j + \sigma_j H_j^{-1}[U_{ij} H_j(x_{ij}'\beta_j)]$. If the marginal distribution of margin $j$ is normal, we can draw the unobserved dependent variable by using $y_{ij}^* = x_{ij}'\beta_j + \sigma_j \Phi^{-1}\left[U_{ij}\Phi\left(\frac{x_{ij}'\beta_j}{\sigma_j}\right)\right]$.

Then, we use the posterior sampling schemes for continuous margins as in Pitt *et al.* (2006) to draw the model parameters.

According to the copula likelihood in Subsection 2.1 and the continuous margins generated from data augmentation technique above, given the complete data the posterior for each model $j$ is given by:

$$p(\theta_j \mid y_j^*, x_j, \theta_{-j}, y_{-j}^*, x_{-j}, R) \propto$$

$$p(\theta_j) \times \prod_{i=1}^{N} |R|^{-1/2} \exp\{\tfrac{1}{2} e_i'(I - R^{-1})e_i\} \prod_{j=1}^{2} h_j(y_{ij}^* \mid x_{ij}; \theta_j),$$

where $-j$ denotes the elements that are not in margin $j$, $p(\theta_j)$ is the prior for $\theta_j$, under the normality assumption on marginal distributions $h_j = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left\{-\frac{(y_{ij}^* - x_{ij}'\beta_j)^2}{2\sigma_j^2}\right\}$, $\boldsymbol{e}_i = (e_{i1}, e_{i2})'$, and $e_{ij} = \frac{y_{ij}^* - x_{ij}'\beta_j}{\sigma_j}$.

With the flat prior, the posterior density is then given by:

$$p(\theta_j \mid \cdot) \propto \left(\frac{1}{\sigma_j^2}\right)^{1/2} \times |R|^{-N/2} \prod_{i=1}^{N} \exp\{\tfrac{1}{2} e_i'(I - R^{-1})e_i\}$$

$$\prod_{j=1}^{2} h_j(y_{ij}^* \mid x_{ij}; \theta_j).$$

It follows that the logarithm of posterior density is given by:

$$\ln p(\theta_j \mid \cdot) = -\frac{\ln \sigma_j^2}{2} - \frac{N}{2}\ln|R| + \frac{1}{2}\left[1 - (R)_{jj}^{-1}\right]\sum_{i=1}^{N} e_{ij}^2$$

$$-\sum_{i=1}^{N}\sum_{k=1,k\neq j}^{2} (R)_{jk}^{-1} e_{ij} e_{ik} + \sum_{i=1}^{N} \ln h_j(y_{ij}^* \mid x_{ij}; \theta_j).$$

For the correlation coefficient $\rho$, the logarithm of posterior density is given by: Since the posterior densities

$$\ln p(R \mid \cdot) \propto -\frac{N}{2}\ln|R| + \frac{1}{2}\left[1 - (R)_{jj}^{-1}\right]\sum_{i=1}^{N} e_{ij}^2$$

$$-\sum_{i=1}^{N}\sum_{k=1,k\neq j}^{2} (R)_{jk}^{-1} e_{ij} e_{ik}.$$

Since the posterior densities are not in closed form, we use the random walk Metropolis-Hastings (MH) algorithm to draw the parameter values from the posterior densities. As for the proposal densities in M-H algorithm, we use the multivariate t distribution with degree of freedom equaling five for the regression coefficients $\beta_j$, inverse-Gamma for the scale parameter $\sigma_j$, and uniform distribution (-1,1) for the correlation coefficient $\rho$. For the draws of regression coefficients, we iterate the log-posterior density to the mode by using a Newton-Raphson method to obtain the location and scale parameters for the multivariate *t* proposal.

We summarize our MCMC procedure as follows:

- Step 0: Set up the initial values for the all model parameters.
- Step 1: Generate $y^*_{i1} \le 0$ from truncated normal distribution and bivariate Gaussian copula through data augmentation specified above.
- Step 2: Sample $\theta_1$ by using M-H algorithm.
- Step 3: Generate $y^*_{i2} \le 0$ from truncated normal distribution and bivariate Gaussian copula through data augmentation.
- Step 4: Sample $\theta_2$ by using M-H algorithm.
- Step 5: Sample $\rho$ by using M-H algorithm.

## 4. Simulation Study

In this section, we perform a simulation study by using a bivariate Gaussian copula with two Gaussian margins. Our Tobit model for each marginal *j* is given by:

$$y_j = \beta_{j,0} + \beta_{j,1}x_{j,1} + \beta_{j,2}x_{j,2} + \beta_{j,3}x_{j,3} + \sigma_j e_j,$$

where $y_j, x_{j,1}, x_{j,2}, x_{j,3}, e_j$ are $N \times 1$ vectors, $\beta_{j,0}, \beta_{j,1}, \beta_{j,2}, \beta_{j,3}, , \sigma_j$ are scalar parameter values, $e_j \sim N(0,1)$. For marginal model 1, we set $\beta_1 = (\beta_{1,0}, \beta_{1,1}, \beta_{1,2}, \beta_{1,3})'$=(-5,1,-1.5,0.1)$'$,$\sigma_1$ = 1, and generate the coveriates $x_{1,1} = 2 \times U_1$,$x_{1,2} = 0$ if $U_2 \le 0.5$ and 1 *otherwise* where $U_1$ and $U_2$ are drawn separately form U*nif*(0,1), and $x_{1,3}$ is a discrete variable from U*nif*(25,60). For marginal model 2, we set $\beta_2 = (\beta_{2,0}, \beta_{2,1}, \beta_{2,2}, \beta_{2,3})' =$ (6,4,3.5,-0.5)$'$,$\sigma_2$ = 2, and generate the covariates $x_{2,1} = 5 \times U_3$, $x_{2,2} = 0$ if $U_4 \le 0.5$ and 1 otherwise where $U_3$ and $U_4$ are drawn separately from U*nif*(0,1), and $x_{2,3}$ is a discrete variable form U*nif*(25,60). We set $\rho$ = 0.1. With $N$ = 500 for each marginal model *j*, we obtain the SUR Tobit model with 62% and 65% degrees of censoring for marginal models 1 and 2, respectively. After 6,000 iterations with 1,000 burn-ins and retaining every tenth draw, we obtain the estimation results shown in Table 1.

**Table 1: Estimation Results from Simulation Study**

| | Marginal Model 1 | | | | Marginal Model 2 | | | | Correlation Coef. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | True Value | Mean | S.D. | | True Value | Mean | S.D. | | True Value | Mean | S.D. |
| $\beta_{1,0}$ | -5.0000 | -5.1412 | 0.2972 | $\beta_{2,0}$ | 6.0000 | 7.2668 | 0.5360 | ρ | 0.1000 | 0.1182 | 0.0657 |
| $\beta_{1,1}$ | 1.0000 | 1.0618 | 0.0832 | $\beta_{2,1}$ | 4.0000 | 3.5750 | 0.1522 | | | | |
| $\beta_{1,2}$ | -1.5000 | -1.3253 | 0.1026 | $\beta_{2,2}$ | 3.5000 | 3.4585 | 0.2097 | | | | |
| $\beta_{1,3}$ | 0.1000 | 0.0998 | 0.0057 | $\beta_{2,3}$ | -0.5000 | -0.4859 | 0.0237 | | | | |
| $\sigma^2_1$ | 1.0000 | 1.0767 | 0.0866 | $\sigma^2_2$ | 4.0000 | 3.9855 | 0.2996 | | | | |

The results indicate that the posterior means and standard deviations for the model parameters are close to the true values. The acceptance rates for the regression coefficients and scale parameter of marginal model 1 are 51% and 81%, respectively. The acceptance rates for marginal model 2 are 58% and 82%, respectively, while the acceptance rate of the correlation coefficient is quite low at 8%. The low acceptance rate of the correlation coefficient is not beyond our expectation since it can be attributed to our use of the uniform distribution as the proposal density. Figures 1 and 2 show the convergence draws and the posterior densities of the model parameters. The convergence draws show that the draws of regression coefficients and scale parameter in both models converge quite well, and better than the correlation coefficient.

**Figure 1: Convergence Draws of Model Parameters**

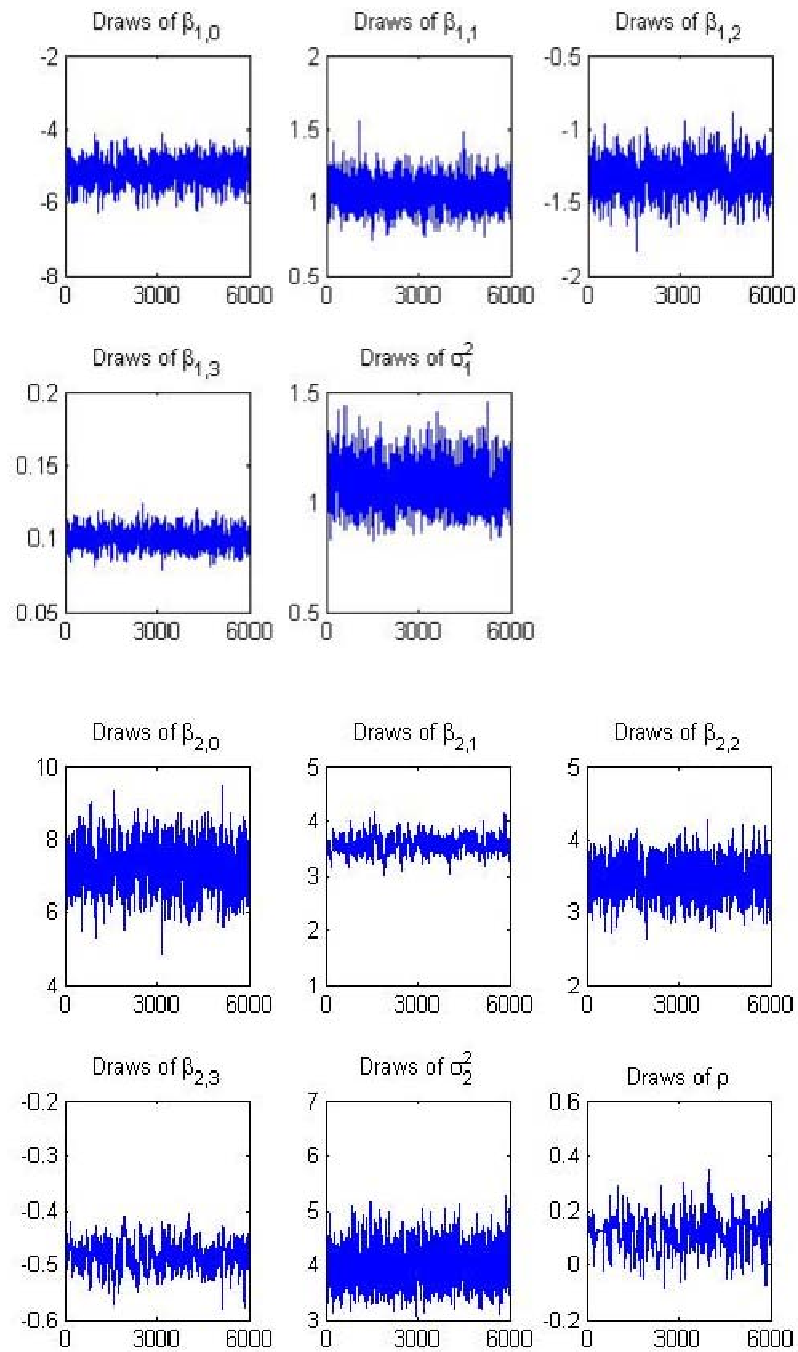
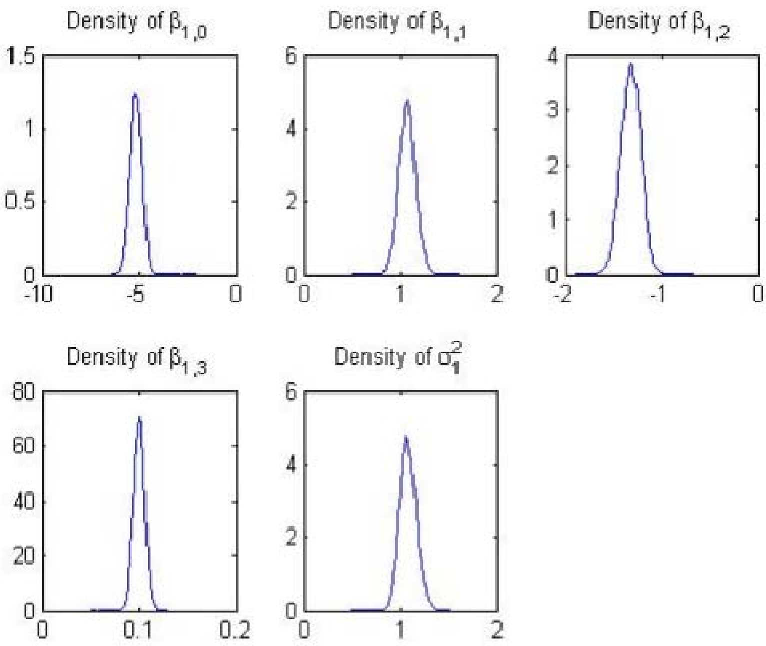
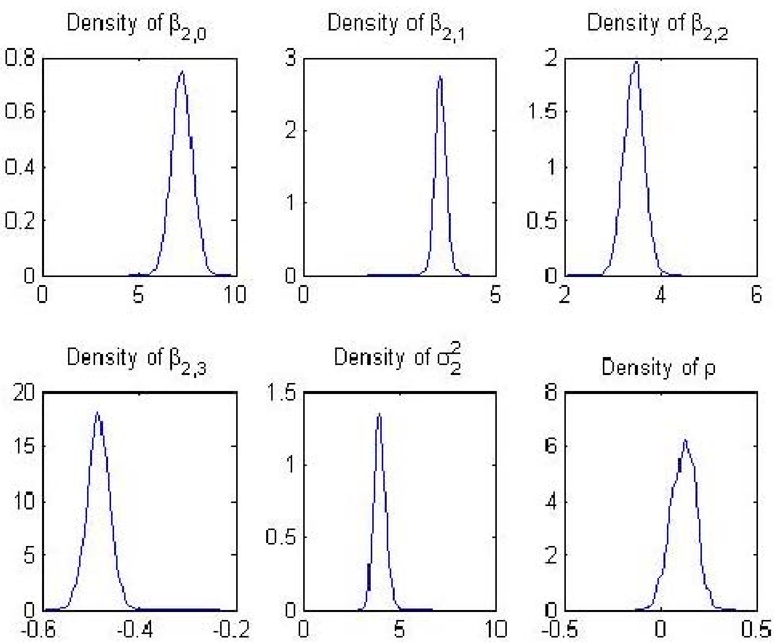



**Figure 2: Posterior Densities of Model Parameters**




## 5. Application

Following the above simulation, we now move on to empirically apply our model and method to Thai wage earnings data from the 2002 Socioeconomic Survey by the National Statistical Office of Thailand. In this application, we determine the correlated errors or the dependence between the wage earnings of the household head and the average wage earnings of household members who are not attending school. Our SUR Tobit model specification is given by:

$$Wage_i^H = \beta_0^H + \beta_1^H Age_i + \beta_2^H Age_i^2 + \beta_3^H School_i + \beta_4^H NonWage_i + \beta_5^H Gender_i + \beta_6^H Central_i + \beta_7^H North_i + \beta_8^H NE_i + u_i^H ,$$

$$\overline{Wage}_i^M = \beta_0^M + \beta_1^M \overline{Age_i} + \beta_2^M \overline{Age_i^2} + \beta_3^M \overline{School_i} + \beta_4^M \overline{NonWage_i} + \beta_5^M Male_i + \beta_6^M Central_i + \beta_7^M North_i + \beta_8^M NE_i + u_i^M ,$$

where i = household i,

$Wage_i^H$ = wage earnings of household head in '000 Baht/month,

$Age_i$ = age of household head in years,

$School_i$ = number of years in schools of household head,

$NonWage_i$ = dummy variable for non-wage earnings, 1 if household head has positive non-wage earnings and 0 otherwise,

$Gender_i$ = gender of household head, 1 if male and 0 if female,

$Central_i$ = dummy variable for household location, 1 if in central region and 0 otherwise,

$North_i$ = dummy variable for household location, 1 if in northern region and 0 otherwise,

$NE_i$ = dummy variable for household location, 1 if in north eastern region and 0 otherwise,

$\overline{Wage}_i^M$ = average wage earnings of household members in '000 Baht/month,

$\overline{Age}_i$ = average age of household members in years,

$\overline{School}_i$ = average number of years in schools of household members,

$\overline{NonWage}_i$ = dummy variable for average non-wage earnings, 1 if household members have positive average non-wage earnings and 0 otherwise,

$Male_i$ = number of male members in the household

$u_i^H$ = error term for household head equation and $u_i^H \sim N(0,\sigma_H^2)$ ,and

$u_i^M$ = error term for household member equation and $u_i^M \sim N(0,\sigma_M^2)$ ,

We expect a positive relationship among age, schooling, male head and members, and all non-South regional dummy variables since wage earnings increase as age and schooling increase. Male head and members have a better chance to gain higher wage earnings than female counterparts. Also, on average, people in other regions receive higher wage earnings than those in the south.

In contrast, we anticipate finding a negative relationship with age squared and non-wage earnings income. The wage earnings as a function of age should be concave, i.e., wage earnings should decrease after some age while non-wage earnings should be considered as a substitute for the wage earnings.

In the estimation, we separate the dataset into urban and rural areas in order to detect the difference in the estimation results between the two areas. The data summary from Table 2 shows that he average household head had wage earnings almost two times higher than those of household members in the urban areas; while the wage earnings gap between the household head and members was narrower in the rural areas. Wage earnings data for both the household head and members are quite symmetric in the urban areas, while they are skewed in the rural areas. All data in all areas exhibited heavy tails, which can be seen from the high kurtosis values. Figures 2 displays the distributions of wage earnings data of the household head vs. members from the urban and rural areas. The degrees of censoring for the wage earnings of household head and members from 2231 households in the urban areas are 55% and 54%, respectively, while they are 58% and 55% from the 1445 households in the rural areas.

**Table 2: Summary of Thai Wage Earnings Data in 2002 ('000 Baht)**

| | Urban | | | | Rural | | | |
|---|---|---|---|---|---|---|---|---|
| | Head | | Members | | Head | | Members | |
| | All Obs. | Positive | All Obs. | Positive | All Obs. | Positive | All Obs. | Positive |
| Mean | 5.133 | 11.431 | 3.102 | 6.719 | 2.271 | 5.423 | 1.634 | 3.611 |
| S.D. | 9.667 | 11.668 | 6.091 | 7.488 | 5.527 | 7.477 | 3.740 | 4.877 |
| Skewness | 3.68 | 2.99 | 3.79 | 2.94 | 5.07 | 3.60 | 4.17 | 2.93 |
| Kurtosis | 21.75 | 14.69 | 28.19 | 19.47 | 38.38 | 19.46 | 22.85 | 11.05 |
| No. of obs. | 2231 | 1002 | 2231 | 1030 | 1445 | 605 | 1445 | 654 |
| % Censoring | 55.1 | n.a. | 53.8 | n.a. | 58.1 | n.a. | 54.7 | n.a. |

Note: n.a.= not available

**Figure 2: Probability Distribution Functions of Thai Wage Earnings Data in 2002**

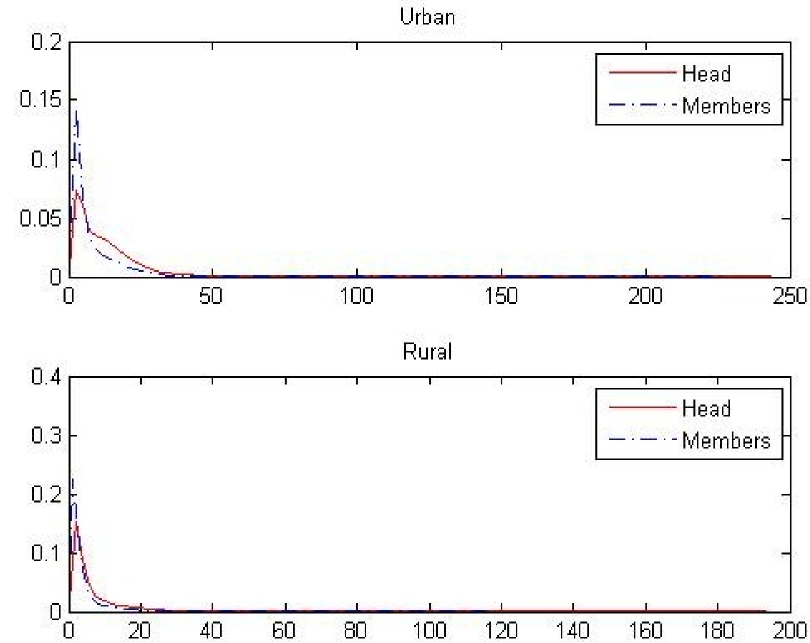



Table 3 shows the results of wage earnings from a SUR Tobit model estimated through MCMC methods for urban households. The regression coefficients of the household head equation have the expected sign. Wage earnings increase as age and schooling, while being a male head can help get higher wage earnings. The negative coefficient on the age-squared term indicates the concavity of the functional form for age while the negative non-wage earnings coefficient confirms the negative relationship or the substitution effect between wage and non-wage earnings. It is worth mentioning that the wage earnings of the household head in other regions are not significantly different from those in southern Thailand. The regression coefficients of the household member equation also have the expected sign, but schooling is the only variable that whose coefficient is significant. The estimated scale parameter of the household head equation ($\sigma^2$) is 3.5 times higher than that of household member equation ($\sigma^2$). The estimated correlation coefficient of 0.48 indicates the significant dependency of wage earnings between household head and members.

**Table 3: Estimation Results of Wage Earnings Equations in Urban Areas**

| Household Head | | | Household Members | | | Correlation Coef. | | |
|---|---|---|---|---|---|---|---|---|
| Variable | Mean | S.D. | Variable | Mean | S.D. | Variable | Mean | S.D. |
| ***Constant*** | -53.5863 | 6.2176 | ***Constant*** | -10.2208 | 1.6971 | $\rho$ | 0.4805 | 0.0227 |
| ***Age$_i$*** | 1.7908 | 0.2429 | $\overline{Age_i}$ | 0.0574 | 0.0747 | | | |
| ***Age$^2_i$*** | -0.0176 | 0.0024 | $\overline{Age_i^2}$ | -0.0001 | 0.0009 | | | |
| ***School$_i$*** | 1.3488 | 0.0743 | **$\overline{School_i}$** | 0.9150 | 0.0677 | | | |
| ***NonWage$_i$*** | -7.5556 | 0.7960 | $\overline{NonWage_i}$ | -0.4809 | 0.3984 | | | |
| ***Gender$_i$*** | 2.9735 | 0.9611 | *Male$_i$* | 0.3761 | 0.2952 | | | |
| *Central$_i$* | 1.5942 | 1.0456 | *Central$_i$* | 1.0683 | 0.6402 | | | |
| *North$_i$* | 0.5723 | 1.0885 | *North$_i$* | 0.6575 | 0.6735 | | | |
| *NE$_i$* | 1.6149 | 1.0411 | *NE$_i$* | -0.0823 | 0.6466 | | | |
| $\sigma^2_H$ | 154.3213 | 6.5430 | $\sigma^2_M$ | 61.6036 | 2.4178 | | | |

Note: Bold font indicates the 5% level of significance

Figures 3 and 4 show the convergence draws and the posterior densities of parameter estimates from wage earnings in urban areas. The convergence draws show that the parameter draws in both equations converge quite well. The acceptance rates for regression coefficients and the scale parameter of household head equation are 99.5% and 70%, respectively; while the acceptance rates are, respectively, 98% and 76% for the case of the household member equation. The acceptance rate for the correlation coefficient is 2.5%.

The estimation results for the rural households from Table 4 show the similar results to those for urban households. The regression coefficients of the household head equation have the expected sign, the same as those of the household member equation except that the coefficient of age squared is positive but insignificant. It is worth mentioning that the number of significant coefficients in the rural household member equation is higher than that of the urban household counterpart. This means that there is a significant difference in household member characteristics between urban and rural areas. Also, in the case of rural areas the estimated scale parameter of the household head equation ($\sigma2$ ) is 2 times higher than that of household member equation ($\sigma2$ ), which is closer than those in the urban areas; while the estimated correlation coefficient of 0.34 in rural households is slightly lower than that in urban households.

Figures 5 and 6 show the convergence draws and the posterior densities of parameter estimates from wage earnings in rural areas. The draws and densities are similar to those in urban households. The acceptance rates for regression coefficients and the scale parameter of the household head equation are 89% and 78%, respectively, while the acceptance rates for household member equation are 47% and 78%. The acceptance rate for the correlation coefficient is 4%.

**Figure 3: Convergence Draws of Model Parameters for Wage Earnings in Rural Areas**

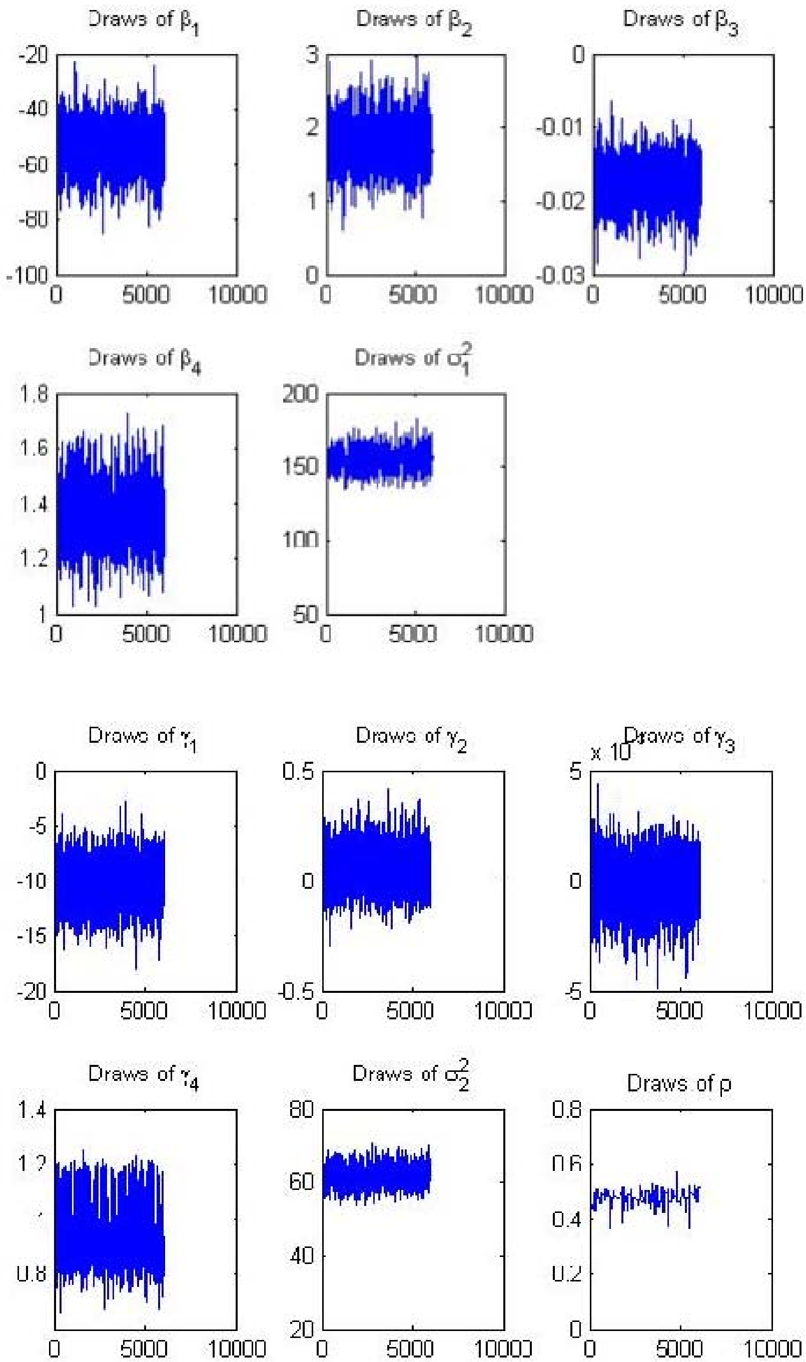


**Figure 4: Posterior Densities of Model Parameters for Wage Earnings in Rural Areas**

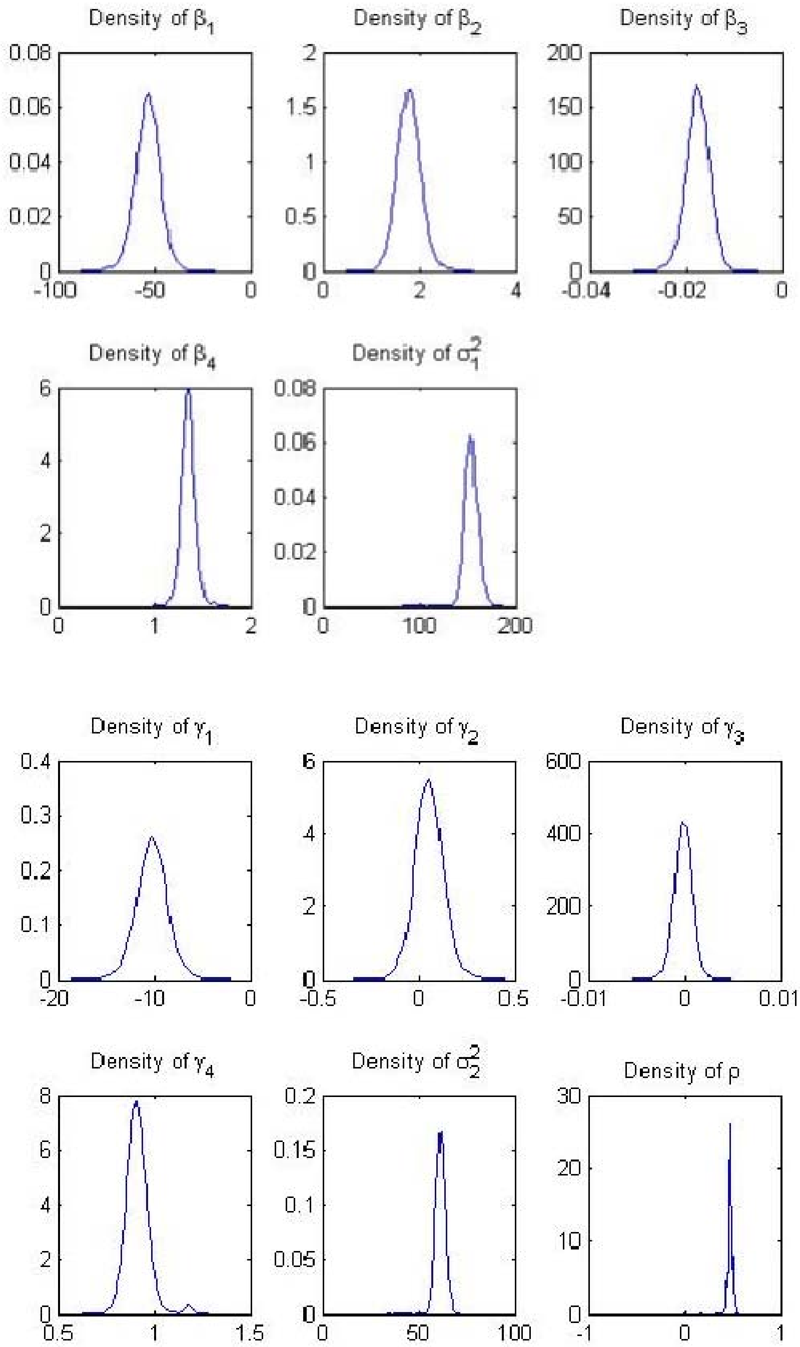


**Table 4: Estimation Results of Wage Earnings Equations in Rural Areas**

| Household Head | | | Household Members | | | Correlation Coef. | | |
|---|---|---|---|---|---|---|---|---|
| Variable | Mean | S.D. | Variable | Mean | S.D. | Variable | Mean | S.D. |
| ***Constant*** | -35.6728 | 6.2221 | ***Constant*** | -7.0851 | 0.7063 | $\rho$ | 0.3435 | 0.0412 |
| ***Age<sub></sub>***$_i$ | 1.1923 | 0.2326 | $\overline{Age_i}$ | 0.0098 | 0.0322 | | | |
| $\boldsymbol{Age^2_i}$ | -0.0120 | 0.0022 | $\overline{Age^2_i}$ | 0.0001 | 0.0004 | | | |
| $\boldsymbol{School_i}$ | 1.1244 | 0.0725 | $\overline{\boldsymbol{School_i}}$ | 0.8948 | 0.0324 | | | |
| $\boldsymbol{NonWage_i}$ | -6.8049 | 0.6268 | $\overline{\boldsymbol{NonWage_i}}$ | -0.8986 | 0.2004 | | | |
| $\boldsymbol{Gender_i}$ | 4.2477 | 0.7823 | $\boldsymbol{Male_i}$ | 0.3501 | 0.1538 | | | |
| $Central_i$ | 0.2259 | 0.6354 | $\boldsymbol{Central_i}$ | 1.3034 | 0.2703 | | | |
| $North_i$ | 1.0053 | 0.6633 | $\boldsymbol{North_i}$ | 1.2280 | 0.2889 | | | |
| $NE_i$ | 1.0563 | 0.6785 | $NE_i$ | 0.2775 | 0.2807 | | | |
| $\sigma^2_H$ | 53.5038 | 2.7102 | $\sigma^2_M$ | 21.1445 | 0.8927 | | | |

Note: Bold font indicates the 5% level of significance

**Figure 5: Convergence Draws of Model Parameters for Wage Earnings in Urban Areas**

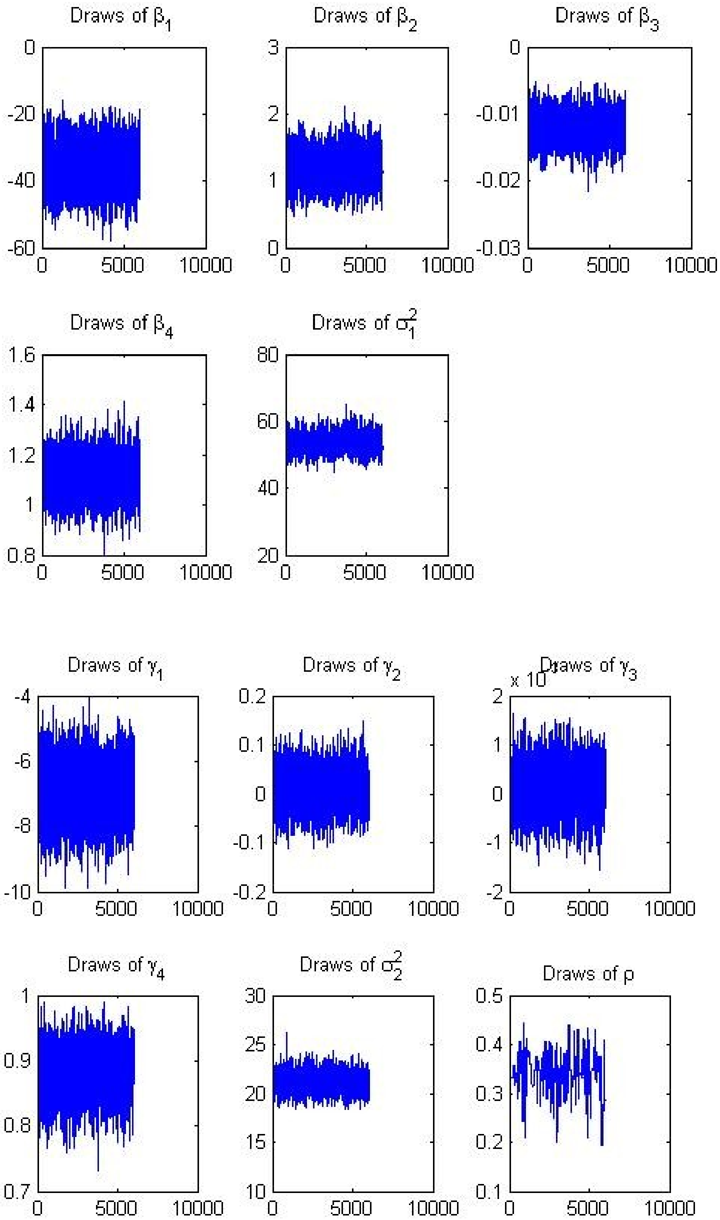
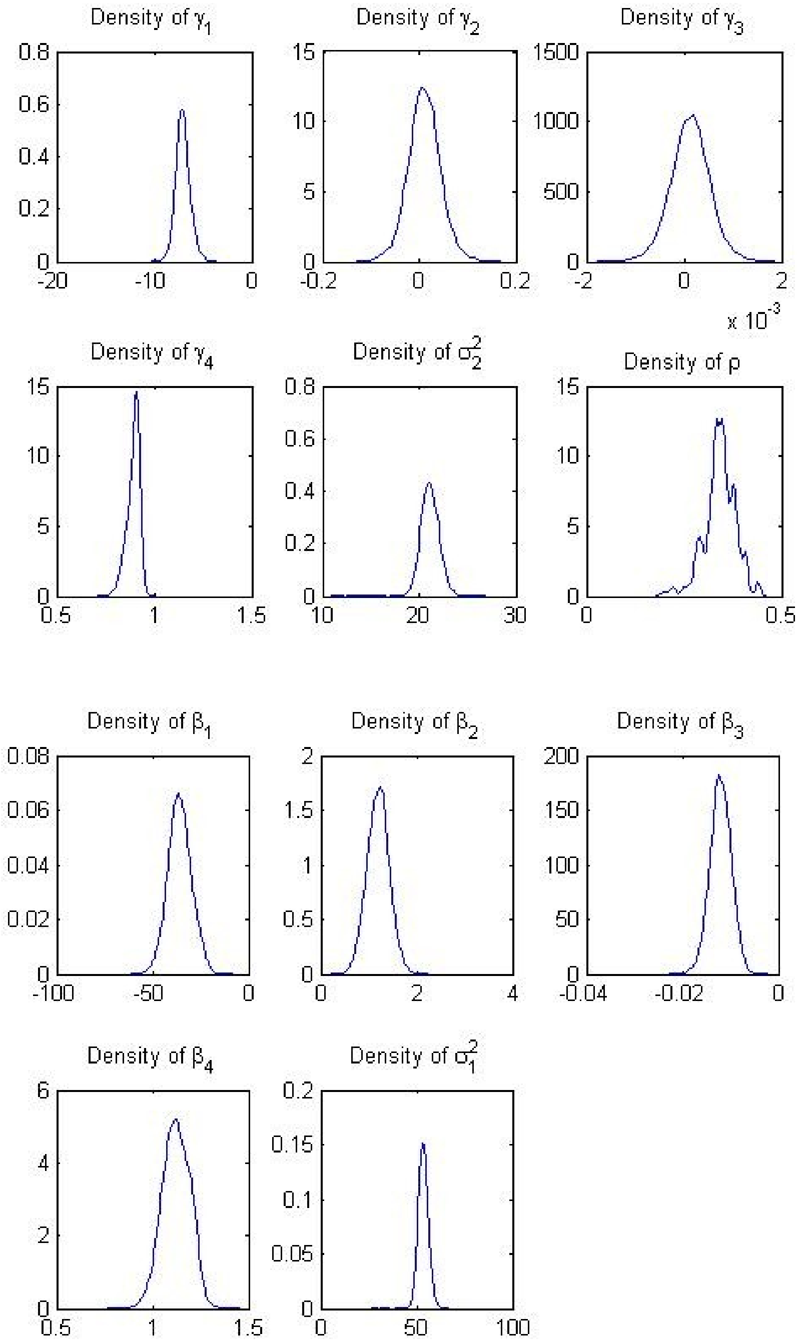


**Figure 6: Posterior Densities of Model Parameters for Wage Earnings in Urban Areas**




## 6. Conclusion

In this paper, we have attempted to model the dependence structure of a SUR Tobit model through a bivariate Gaussian copula. The copula permits flexibility in joining different marginal distributions. By using the MCMC and data augmentation techniques, we have estimated the SUR Tobit model through a bivariate Gaussian copula with Gaussian margins and obtained satisfactory results in both the simulation and empirical phases of this research. For the empirical study, we have applied our model and methods to Thai wage earnings data in 2002, revealing a significant dependency of wage earnings between the household head and members in both urban and rural areas.

**References**

Brown, E. and Lankford, H., 1992, Gifts of money and gifts of time: Estimating the effects of tax prices and available time. Journal of Public Economics 47, 321-341.

Chib, S., 1992, Bayes regression for the Tobit censored regression model. Journal of Econometrics 51, 79-99.

Greene, W.H., 2003, Econometric Analysis. 5th edition, New Jersey: Prentice Hall.

Huang, C.J., Sloan, F.A. and Adamache, K.W., 1987, Estimation of seemingly unrelated Tobit regressions via the EM algorithm. Journal of Business and Economic Statistics 5(3), 425-430.

Huang, H.-C., 1999, Estimation of the SUR Tobit model via the MCECM algorithm. Economics Letters 64, 25-30.

Huang, H.-C., 2001, Bayesian analysis of the SUR Tobit model. Applied Economics Letters 8, 617-622.

Kamakura, W.A. and Wedel, M., 2001, Exploratory Tobit factor analysis for multivariate censored data. Multivariate Behavioral Research 36(1), 53-82.

McNeil, A.J., Frey, R. and Embrechts, P., 2005, Quantitative Risk Management: Concepts, Techniques and Tools. New Jersey: Princeton University Press.

Meng, X. and Rubin, D.B., 1996, Efficient methods for estimation and testing with seemingly unrelated regressions in the presence of latent variables and missing observations. In Bayesian Analysis in Statistics and Econometrics, Berry, D.A., Chaloner, K.M. and Geweke, J.K., (eds.,. Wiley and Sons, 215-227.

Nelsen, R.B., 2006, An Introduction to Copulas. 2nd edition. New York: Springer.

Patton, A., 2009, Copula-based models for financial time series. In Handbook of Financial Time Series, Andersen, T.G., Davis, R.A., Kreiss, J.-P. and Mikosch, T., (eds.), Berlin: Springer Verlag, 767-785.

Pitt, M.,D. Chan, R.K., 2006, Efficient Bayesian inference for Gaussian copula regression models. Biometrika 93(3), 537-554.

Sklar, A., 1959, Fonctions de repartition a n dimensions et leurs marges. Publ. Inst. Statist. Univ. Paris. 8, 229-231.

Song, P.X.-K., 2000, Multivariate dispersion models generated from Gaussian copula. Scandinavian Journal of Statistics 27, 305-320.

Song, P.X.-K., Li, M. and Yuan, Y., 2009, Joint regression analysis of correlated data using Gaussian copulas. Biometrics 65, 60-68.

Tanner, M.A. and Wong, W.-H., 1987, The calculation of posterior distributions by data augmentation (with discussion). Journal of the American Statistical Association 82, 528-550.

Taylor, M.R. and Phaneuf, D., 2009, Bayesian Estimation of The Impacts of Food Safety Information on Household Demand for Meat and Poultry. Paper presented at the Agricultural & Applied Economics Association's 2009 AAEA & ACCI Joint Annual Meeting, Milwaukee, WI, July 26-28, 2009.

Trivedi, P.K. and Zimmer, D.M., 2005, Copula modeling: An introduction for practitioners. Foundations and Trends in Econometrics 1(1), 1-111.

Wales, T.J. and Woodland, A.D., 1983, Estimation of consumer demand systems with binding non-negativity constraints. Journal of Econometrics 21, 263-285.

Zellner, A., 1962, An efficient method of estimating seemingly unrelated regressions and tests for aggregate bias. Journal of the American Statistical Association 57, 348-368.