The stable relationship between crude oil price and petrol price: 
Evidence from multivariate GARCH model

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ABSTRACT

This study investigates the relationship between crude oil price and petrol price as well as their behaviour using daily U.S. price series in the period from January 11th 1988 to May 20th 2011. We find that univariate GARCH(1,1) is likely the most suitable model to measure the volatility of relative changes in the crude oil price and the petrol price. Whereas, a multivariate GARCH(1,1) model of the diagonal BEKK type is employed to analyse the conditional correlation of the two series. Although the view of asymmetrical responses in the crude oil price and the petrol price is unsupported by this study, it is evident that there is a strongly positive correlation between them in the long-run.

Keywords: ARCH effect, GARCH, BEKK, Oil price, Petrol price

JEL Classification: D43, Q4.
1. Introduction and brief literature review

The behaviour of the oil price and petrol price and the relationship between them are hotly debated issues in the recent literature of economics (Bachmeier & Griffin, 2003; Balaguer & Ripollés, 2012; Radchenko, 2005; Radchenko & Shapiro, 2011). Manning (1991) confirms that there is a steady relationship between petrol price and oil price by using an Error-correction mechanism (ECM) model to analyse the U.K. data during the period 1973-1988. His study also provides empirical evidence for the existence of asymmetric responses in petrol price in the short-run. Employing ECM model to study data from Northwest European market in the period from 1990 to 2005, Frewer (2006) supports the view that the prices of oil and its products are co-integrated in the long-run. Bettendorf, Van der Geest, and Kuper (2005) estimates an EGARCH model for Dutch retail gasoline price and shows that although no amount asymmetry exists in long-run, this phenomenon appears in short-run.

Whereas, through the standard Engle-Granger two-step estimation procedure, Bachmeier and Griffin (2003) run an error-correction model with the U.S. daily spot petrol and crude oil price data over the period 1985-1998 and find no evidence of asymmetry in wholesale gasoline prices. Similarly, Balaguer and Ripollés (2012) investigate the relationship between international wholesale oil prices to Spanish retail fuel prices by employing a GARCH(1,1) process. Their research’s result supports the proposition that “the short-term transmission of wholesale prices to retail prices is quite symmetric for both gasoline and diesel fuel” (Balaguer & Ripollés, 2012, p. 2066).

In line with several previous studies, the current study aims at exploring the behaviour of oil price and petrol price as well as their relationship by using daily U.S. data on these two variables. We use U.S. data for two reasons: (i) the data is available and adequate allowing us to better adopt potential estimations, and (ii) using U.S. data facilitates comparing our study findings with those of previous studies. Unlike previous studies, a multivariate GARCH model of diagonal BEKK type will be employed to analyse the correlation of oil price and petrol price in long-run.

The remainder of the paper is structured as follows. Section 2 will describe data sources and summarize ARCH model and the formal tests of ARCH-effects. The univariate and multivariate GARCH models will then be briefly introduced to lay the theoretical foundation of applications in the next section. The application of the above models on exploring the behaviour of the two series will be presented in section 3. GARCH(1,1) model is employed to investigate the behaviour of the conditional variance of \( x \) series on the basis of using an ARMA(1,1) in mean equation. The variance of the crude oil price will then be forecasted. The volatility of \( y \) series will be measured by GARCH(1,1) model with an AR(1) specification in mean equation. Besides, the diagonal BEKK Multivariate GARCH model will also be estimated to evaluate the behaviour of the conditional correlation coefficient between these series. Finally, the conclusion section will provide an overall assessment of our findings about the behaviour and relationship between oil price and petrol price in the long run.
2. Data and Method

2.1. Data
The current study employs daily U.S. price series of petrol and crude oil in the period from January 11th 1988 to May 20th 2011, with 6095 observations on each series. The petrol prices are taken from the S&P GSCI Unleaded Gasoline Spot price index, while the crude oil prices are WTI Spot Cushing US$/BBL, downloaded from Datastream. These raw data are then transformed into natural logarithm form, denoted as \( l_{petrol} \) and \( l_{oil} \). We define that \( y \) is the series of changes in \( l_{petrol} \) and \( x \) is the series of changes in \( l_{oil} \), respectively expressed in formulas (1) and (2) as follows:

\[
y_i = l_{petrol_i} - l_{petrol_{i-1}} \quad (1)
\]

\[
x_i = l_{oil_i} - l_{oil_{i-1}} \quad (2)
\]

Damodar (2004) notes that “changes in the log of a variable denote relative changes, which, if multiplied by 100, give percentage changes”. This means that:

\[
y_i = l_{petrol_i} - l_{petrol_{i-1}} = \ln\left(\frac{petrol_i}{petrol_{i-1}}\right) \approx \frac{petrol_i - petrol_{i-1}}{petrol_{i-1}} \quad (3)
\]

\[
x_i = l_{oil_i} - l_{oil_{i-1}} = \ln\left(\frac{oil_i}{oil_{i-1}}\right) \approx \frac{oil_i - oil_{i-1}}{oil_{i-1}} \quad (4)
\]

Thus, in this case, investigating the behaviour of \( x \) and \( y \) is equivalent to those of the relative changes of the crude oil price and the petrol price over time.

2.2. Methods and model specifications
Damodar (2004) and Brooks (2008) state that the two non-linear models, which are widely used in the domain of finance to estimate and predict volatility, are ARCH and GARCH models. In line with several prior studies on the behaviour of oil and petrol price, we use univariate and multivariate GARCH models in this study. We believe that employing multi-model approach will enables us to select the most suitable estimations for the two series as well as the relationship between them. The following content is quoted and summarized from (Brooks, 2008; Damodar, 2004; Verbeek, 2004).

2.2.1. Autoregressive conditionally heteroskedastic (ARCH) models
Suppose that we employ AR(1) specification in mean equation, so that an ARCH(q) model, where the error variance depends on \( q \) lags of squared errors, would be written as mean equation (5) and conditional variance equation (6) as follows.

\[
y_i = \mu + \phi y_{i-1} + u_i, \quad u_i \sim N(0, \sigma_i^2) \quad (5)
\]

\[
\sigma_i^2 = \alpha_0 + \alpha_1 u_{i-1}^2 + \alpha_2 u_{i-2}^2 + \ldots + \alpha_q u_{i-q}^2 \quad (6)
\]
2.2.2. Testing for ARCH effects

According to Brooks (2008), testing for ARCH effects should be conducted before applying an appropriate GARCH models to ensure whether we should use GLS method instead of OLS method. The test procedure includes the following main steps:

(i) Estimate the mean equation of $x$ and $y$ series by OLS method. The explanatory variables in these equations could be lagged dependent variable and/or other exogenous variables that impact on dependent variable. The residuals are then saved.

(ii) Square the residuals and regress them on $q$ lags to test for ARCH of order $q$, as equation (7), and obtain $R^2$ from this regression.

$$
\hat{\gamma}_1^2 + \gamma_2 \hat{\gamma}_2^2 + \ldots + \gamma_q \hat{\gamma}_q^2 + \nu_i
$$

(iii) The join null hypothesis of no ARCH effects in the residuals ($\gamma_1 = 0$ and $\gamma_2 = 0 \ldots$ and $\gamma_q = 0$) will be rejected if the test statistic $T.R^2$ is greater than the critical value from the $\chi^2$ distribution (where: $T$ is the number of observations).

2.2.3. Univariate Generalised ARCH (GARCH) models

Brooks (2008) and Holmes (2011) document that GARCH models could be used to capture the volatility clustering effects in time series, and that they are more parsimonious than ARCH model. The GARCH models allow the conditional variance to be dependent upon previous own lags. The conditional variance equations for GARCH(1,1) ; GARCH(2,2) ; and GARCH(2,1) model, which will be used in the next section, are expressed in equations (8), (9), and (10):

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2
$$

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2
$$

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2
$$

In this study, we also employ two other asymmetric GARCH-type models, designed to capture leverage effects, to investigate the behaviour of time series, including Threshold-GARCH model and Exponential-GARCH model. The conditional variance equations for TGARCH(1,1) and EGARCH(1,1) models are equations (11) and (12) respectively.

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + \gamma_1 u_{t-1}^2 I_{t-1}
$$

where $I_{t-1} = 1$ if $u_{t-1} \leq 0$

$L_{t-1} = 0$ otherwise

$$
\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \left[ \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] + \alpha_2 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta_1 \ln(\sigma_{t-1}^2)
$$
2.2.4. Multivariate GARCH models – The BEKK model

The BEKK is one class of the multivariate GARCH models, which are very similar to univariate models. The most vital characteristic of this model is that it ensures positive definition of $H_t$ matrix by its quadratic nature (Brooks, 2008). The BEKK model is formulated by equation (13).

$$H_t = W'W + A'H_{t-1}A + B'E_{t-1}E_{t-1}'B$$

Where: $H_t$ is a 2x2 conditional variance-covariance matrix; $E_t$ is a 2x1 disturbance vector; $A$ and $B$ are 2x2 matrices of parameters; and W is an upper triangular matrix of parameters.

3. Results and Discussion

3.1. Data description

As we can observe from Figure 1, the relative changes of crude oil price show the periods of wide swings and the other periods of rather calm. The story is the same for oil series (Figure 2). The volatility pattern of the relative changes of petrol price is very similar to that of crude oil price. This implies the phenomenon of clustering volatility in these time series.

![Figure 1. Time series of the relative changes of crude oil price (x)](image)

It is noted that the volatility of the two series in the periods of 1990-1991 and 2008-2009 are extremely larger than that in the other periods. It is likely related to the Gulf War and the world economic crisis in these periods in turn. Moreover, intuitively, the stationary of these series is depicted clearly in the two figures. The correlogram and the formal unit root tests (unreported to save space) provide statistical support to the stationary of the two time series.
3.2. Test for the presence of ARCH-effects

This study begins with estimating a postulated linear regression of AR(1) on x and y series by OLS method to obtain the residuals. Then each series of residuals will be employed to test for the presence of ARCH effects in the behaviour of x and y series. Table 1 and Table 2 show the results of the Lagrange multiplier test for ARCH (with 3 lags) for the two series. As we can see, both F-statistic and LM-statistics (T.R²) are very significant, suggesting the presence of ARCH in the two series.

Table 1: Test results for ARCH effects in x series

<table>
<thead>
<tr>
<th>Heteroskedasticity Test ARCH</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F-statistic</td>
<td>107.3908</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>306.1811</td>
</tr>
</tbody>
</table>

Table 2: Test results for ARCH effects in y series

<table>
<thead>
<tr>
<th>Heteroskedasticity Test ARCH</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F-statistic</td>
<td>88.55933</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>254.7342</td>
</tr>
</tbody>
</table>

The empirical test results also imply that GARCH-type models may be suitable for the data, and that we should use GLS procedure instead of OLS method to estimate the parameters of the required models.

3.3. GARCH model for x series and prediction of the variance of x

At the beginning, an AR(1) specification is employed in mean equation of x series. However, the coefficient of $x_{t-1}$ is insignificant. We use add-ins ‘Automatic ARIMA selection’ on econometric software EViews 7.1 to confirm that ARMA(1,1) is likely more appropriate for the data (unreported to save space). Thus, the ARMA(1,1)-GARCH(1,1) model is estimated again. Model (8) of Table 3 provides us with estimated parameters of the conditional variance equation that are all highly statistically significant. In order to find out the most appropriate
As we can observe from model (9), the coefficient of the conditional variance term at lag one of GARCH(2,2) is negative and statistically insignificant. Model (10) shows that GARCH(2,1) is also inappropriate for the data because the coefficient of the second lagged squared residual is unsatisfied the non-negativity constraints. To account for possible asymmetric response of volatility to positive and negative shocks in the price of crude oil, we employ two asymmetric GARCH models, including TGARCH and EGARCH.

Model (11) shows that the asymmetry term, $\gamma$, has negative sign and is insignificant at 5% level, which implies that there are no leverage effects in $x$ series. Constrains for non-negativity will be $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$, and $\alpha_1 + \gamma \geq 0$. This implies that the model is still accepted, even if $\gamma = < 0$, provided that $\alpha_1 + \gamma \geq 0$ (Brooks, 2008, p. 405). We can see that $\gamma = -0.010 < 0$ but $\alpha_1 + \gamma = 0.067 + (-0.010) = 0.057 > 0$. It is noted that $\gamma$ must be positive for a leverage effect. Hence, the view that a negative shock to oil price causes larger volatility than a positive shock of the same magnitude does is unsupported by this finding.

Using EGARCH regression, we expect that if the asymmetric response of volatility to shocks is existent in the behaviour of the $x$ series, the asymmetry term, $\alpha_2$, of model (12) will be negative and statistically significant. The unexpected result can be seen in the last row of Table 3. Thus, it is not evident from this model that there are asymmetric responses of volatility to positive and negative shocks in the price of crude oil.

The above empirical analyses support choosing GARCH(1,1) as the most appropriate for $x$ series. To prepare for conditional variance forecasting, the study period is truncated by 4 days so that the sample of the period from January 11th 1988 to May 16th 2011 will be used to run GARCH(1,1). We then employ this estimated regression to ex-post forecast the volatility for the remaining days (from May 17th 2011 to May 20th 2011) and ex-ante forecast for the next day May 23rd 2011 (out-of-sample forecast).

GARCH model for the data, we run several other GARCH-type models. The regression results are shown in Table 3.

Table 3: The GARCH-type models for $x$ series

<table>
<thead>
<tr>
<th>Models</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td>4.17E-06</td>
<td>0.061</td>
<td>–</td>
<td>0.935</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(9.115)</td>
<td>(21.662)</td>
<td>–</td>
<td>(406.916)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(9)</td>
<td>8.34E-06</td>
<td>0.060</td>
<td>0.062</td>
<td>-0.032</td>
<td>0.903</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(8.144)</td>
<td>(15.384)</td>
<td>(16.974)</td>
<td>(-0.769)</td>
<td>(23.908)</td>
<td>–</td>
</tr>
<tr>
<td>(10)</td>
<td>3.97E-06</td>
<td>0.077</td>
<td>-0.018</td>
<td>0.937</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(8.864)</td>
<td>(8.890)</td>
<td>(-1.963)</td>
<td>(356.185)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(11)</td>
<td>4.12E-06</td>
<td>0.067</td>
<td>–</td>
<td>0.935</td>
<td>-0.010</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(8.843)</td>
<td>(17.360)</td>
<td>–</td>
<td>(395.422)</td>
<td>(-1.862)</td>
<td>–</td>
</tr>
<tr>
<td>(12)</td>
<td>-0.147</td>
<td>0.136</td>
<td>-0.003</td>
<td>0.994</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(-25.930)</td>
<td>(27.680)</td>
<td>(-0.921)</td>
<td>(1551.561)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: z-statistics are presented in parentheses

The above empirical analyses support choosing GARCH(1,1) as the most appropriate for $x$ series. To prepare for conditional variance forecasting, the study period is truncated by 4 days so that the sample of the period from January 11th 1988 to May 16th 2011 will be used to run GARCH(1,1). We then employ this estimated regression to ex-post forecast the volatility for the remaining days (from May 17th 2011 to May 20th 2011) and ex-ante forecast for the next day May 23rd 2011 (out-of-sample forecast).
Equation (14) introduces the estimated result for ARMA(1,1)-GARCH(1,1) with z-statistics in parentheses, using truncated data. The coefficients on both $u_{i-1}^2$ and $\sigma_{i-1}^2$ terms in the conditional variance equation are highly significant.

\[ x = 0.0003 + 0.8537x_{i-1} - 0.8816u_{i-1} + u_i \]
\[ (1.586) (16.453) (-18.870) \]
\[ u_i \sim N(0, \sigma^2) \] 
\[ \sigma_i^2 = 4.20E-06 + 0.062u_{i-1}^2 + 0.934\sigma_{i-1}^2 \]
\[ (9.118) (21.692) (405.355) \]

It is noted that the sum of the coefficients on both $u_{i-1}^2$ and $\sigma_{i-1}^2$ terms approximates to one ($0.062 + 0.934 = 0.996$), indicating rather high persistence in the conditional variances. The mean value of the conditional variance, which reflects unconditional variance, is computed by equation (15).

\[ Var(u_i) = \frac{\alpha_0}{1-(\alpha_1 + \beta)} \approx 0.0001076 \] 

\[ \alpha_0 = 0.0003 \]
\[ \alpha_1 = 0.8537 \]
\[ \beta = -0.8816 \]

Figure 3 depicts the behaviour of conditional variances of $x$ series. The volatility of the relative changes in crude oil price is extremely high in the periods of 1990-1991 and 2008-2009, which is consistent with our intuitive observation mentioned in the subsection 3.1.
Figure 4: Dynamic forecast of conditional variances for $x$ series

Figure 4 shows the dynamic forecast of conditional variances for $x$ series which tend to converge upon their long-term average value of the variance (around 0.001076) as the prediction horizon increases.

Table 4: The conditional mean and variance forecast of $x$ series

<table>
<thead>
<tr>
<th>Date</th>
<th>$X$</th>
<th>$x^f$</th>
<th>$\hat{\mu}_t$</th>
<th>Var$^f$</th>
<th>Sd$^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/16/2011</td>
<td>-0.02315</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ex-post forecast</td>
<td>5/17/2011</td>
<td>-0.00474</td>
<td>0.00229</td>
<td>-0.007029</td>
<td>0.0008698</td>
</tr>
<tr>
<td></td>
<td>5/18/2011</td>
<td>0.032387</td>
<td>0.00200</td>
<td>0.030385</td>
<td>0.0008706</td>
</tr>
<tr>
<td></td>
<td>5/19/2011</td>
<td>-0.01672</td>
<td>0.00175</td>
<td>-0.018476</td>
<td>0.0008714</td>
</tr>
<tr>
<td></td>
<td>5/20/2011</td>
<td>0.01061</td>
<td>0.00154</td>
<td>0.009069</td>
<td>0.0008721</td>
</tr>
<tr>
<td>Ex-ante forecast</td>
<td>5/23/2011</td>
<td>NA</td>
<td>0.00136</td>
<td>NA</td>
<td>0.0008729</td>
</tr>
</tbody>
</table>

For the static prediction, as we can see from Table 4, the price of crude oil on May 17th 2011 will increase about 0.23%, with standard deviation at about 2.95%. On May 23rd 2011, crude oil price will rise approximately 0.14% in comparison with the back-end of previous week’s price level, with the standard deviation at about 2.95%. It is worth noting that:

$$\text{var}(x_t | x_{t-1}, x_{t-2}...) = \text{var}(u_t | u_{t-1}, u_{t-2}...)$$ \hspace{1cm} (16)

So by using GARCH model, the forecasts of $\sigma_t^2$ will be the forecasts of the future variance of $x_t$ (Brooks, 2008).
3.4. GARCH model for y series

The similar procedure is conducted for the series of relative changes in petrol price (y series). Coefficient estimates for each of (8), (9), (10), (11), and (12) models using y series data are given in Table 5. Model (9) GARCH(2,2) is unsuitable for the data due to the fact that the estimated coefficients $\alpha_2$ and $\beta_2$ are negative. Model (10) GARCH(2,1) cannot be accepted as well because the estimated coefficients $\alpha_2$ is negative and insignificant. As we can observe from the last two rows of Table 5, the empirical estimations from models (11) and (12) do not provide us with any evidence for the presence of leverage effects in y series. Therefore, it is hard for us to believe that a negative shock to petrol price causes larger volatility than a positive shock of the same magnitude does.

Table 5: The GARCH-type models for y series

<table>
<thead>
<tr>
<th>Models</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td>4.82E-06</td>
<td>0.059</td>
<td>-</td>
<td>0.932</td>
<td>(219.335)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(5.842)</td>
<td>(18.835)</td>
<td></td>
<td>(219.335)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9)</td>
<td>6.05E-07</td>
<td>0.085</td>
<td>-0.076</td>
<td>1.719</td>
<td>-0.729</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(1.982)</td>
<td>(10.131)</td>
<td>(-11.581)</td>
<td>(21.236)</td>
<td>(-9.500)</td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td>4.70E-06</td>
<td>0.068</td>
<td>-0.010</td>
<td>0.933</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(5.730)</td>
<td>(6.750)</td>
<td>(-0.918)</td>
<td>(197.793)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11)</td>
<td>4.68E-06</td>
<td>0.064</td>
<td></td>
<td>0.933</td>
<td>-0.010</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(5.757)</td>
<td>(6.750)</td>
<td></td>
<td>(197.793)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(12)</td>
<td>-0.178</td>
<td>0.123</td>
<td>0.003</td>
<td>0.989</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(-10.538)</td>
<td>(21.483)</td>
<td>(0.828)</td>
<td>(541.828)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: z-statistics are presented in parentheses

The above analyses show that GARCH(1,1) is likely the most appropriate model for y series. The estimated result for AR(1)-GARCH(1,1) with z statistics in parentheses is rewritten in equation (16).

$$y = 0.0002 + 0.0353y_{t-1} + u_t$$

(0.7679) (2.6214)

$$u_t \sim \mathcal{N}(0, \sigma^2)$$

$$\sigma^2_t = 4.82E-06 + 0.0593u^2_{t-1} + 0.9315\sigma^2_{t-1}$$

(5.8419) (18.8348) (219.3351)

The coefficients on both $u^2_{t-1}$ and $\sigma^2_{t-1}$ terms in the conditional variance equation are highly significant. The sum of the coefficients on these terms is fairly close to one, implying that shocks to the conditional variance will be highly unrelenting. The conditional variances converge upon their long-term mean value (equals to 0.000522), which reflects the unconditional variance, as the time horizon rises (Figure 5).
Figure 5: Conditional variances of $y$ series

Figure 6 is a simultaneous depiction of conditional variances of $x$ and $y$ series, indicating a similar pattern of volatility of the relative changes in crude oil and petrol price series. It is likely that the volatility of the relative changes in petrol price is smoother than that in crude oil price.

Figure 6: Conditional variances of $x$ and $y$ series

3.5. Diagonal BEKK multivariate GARCH model for $x$ and $y$ series

In this section, we employ a multivariate GARCH(1,1) model of the diagonal BEKK type {that is, model (13)} to analyse further conditional correlation of $x$ and $y$ series over time. The estimated results are given in Table 6. From Table 6, the estimated conditional variance and covariance equations can be rewritten as three following equations (17), (18), and (19) of which all estimated coefficients are statistical significant at 1% level. This implies the dynamic structure in the conditional variance and covariance equations is strong.
where:

- \( h_{11t} \): the conditional variances at time \( t \) of the \( x \) series
- \( h_{22t} \): the conditional variances at time \( t \) of the \( y \) series
- \( h_{12t} \): the conditional co-variances between the two series at time \( t \)

As we can observe, the degree of persistence in the conditional variance (given by \( \alpha_1 + \beta \)) are large for both equations (17) and (18) implying the degree of clustering volatility in \( x \) and \( y \) series. It is noted that the unconditional covariance between the two series, which can be calculated by estimated parameters form equation (19), is positive.

Table 6: The diagonal BEKK model for \( x \) and \( y \) series

<table>
<thead>
<tr>
<th>Covariance specification: BEKK</th>
<th>( GARCH = M + A1^*RESID(-1)^*RESID(-1)^*A1 + B1^*GARCH(-1)^*B1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M ) is an indefinite matrix</td>
<td>( A1 ) is diagonal matrix</td>
</tr>
<tr>
<td>( B1 ) is diagonal matrix</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transformed Variance Coefficients</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M(1,1) )</td>
<td>7.42E-06</td>
<td>5.15E-07</td>
<td>14.41515</td>
<td>0.0000</td>
</tr>
<tr>
<td>( M(1,2) )</td>
<td>5.75E-06</td>
<td>4.93E-07</td>
<td>11.65342</td>
<td>0.0000</td>
</tr>
<tr>
<td>( M(2,2) )</td>
<td>7.67E-06</td>
<td>6.74E-07</td>
<td>11.37546</td>
<td>0.0000</td>
</tr>
<tr>
<td>( A1(1,1) )</td>
<td>0.293771</td>
<td>0.003959</td>
<td>74.19631</td>
<td>0.0000</td>
</tr>
<tr>
<td>( A1(2,2) )</td>
<td>0.259417</td>
<td>0.004972</td>
<td>52.17798</td>
<td>0.0000</td>
</tr>
<tr>
<td>( B1(1,1) )</td>
<td>0.953691</td>
<td>0.000907</td>
<td>1051.724</td>
<td>0.0000</td>
</tr>
<tr>
<td>( B1(2,2) )</td>
<td>0.958587</td>
<td>0.001494</td>
<td>641.8018</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

We can now consider the long-term relationship between \( x \) and \( y \) series through plotting their conditional correlation coefficients over time, which are derived from the above empirical estimation of BEKK model given in Figure 7. This figure obviously shows that conditional correlations between the relative changes in crude oil price and that in petrol price are strongly positive and highly volatile. This implies that the price of petrol tends to move in the same direction as the price of crude oil over the study period, and that the petrol price exhibits a long-term relationship with the crude oil price. In this study, the econometric evidence powerfully supports the view that there are “volatility spillovers” between the price of crude oil and petrol. This implies that volatility to change in petrol price following a change in volatility of crude oil price. Our study’s findings, therefore, support the argument of Brooks (2008, p. 428) that there is “a tendency for volatility to change in one market or asset following a change in volatility of another”.
4. Conclusion

Using the univariate GARCH(1,1) model, we find no empirical evidence for asymmetrical responses in crude oil and petrol price. Hence, this paper does not support the view that a negative shock to the two products’ price causes larger volatility than a positive shock of the same magnitude does. Although such a finding is inconsistent with several previous research results, it reveals some interesting characteristics of the crude oil and petrol price’s behaviour that need to be considered for future research. As early mentioned, empirical results based on a multivariate GARCH(1,1) of the diagonal BEKK type model confirm that there is a long-run relationship between the crude oil price and petrol price, and that they tend to move in the same direction together over time. This finding provides further empirical evidence to support the viewpoint of the stable relationship between the crude oil and petrol price which is suggested by Frewer (2006), and Manning (1991) and others.

Acknowledgement

The author wishes to thank Professor Mark Holmes (Department of Economics, Waikato Management School, The University of Waikato, New Zealand) for his instruction and data. Any remaining errors are my own.
REFERENCES


